Risk estimation in the thinly traded JSE environment

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Recent evidence has come to the fore which suggests that the major source of bias in the estimation of beta coefficients on the JSE can be attributable to the thinly traded phenomenon. In this paper the suitability of a beta estimation procedure which corrects for the effects of thin trading is investigated for JSE stocks. The investigation reveals that implementation of the correction procedure results in substantial improvements in beta estimation. Furthermore, on the basis of the empirical investigation, suggestions are made regarding the parameters that should be included in the beta estimator.

Onlangse getuienis dui daarop dat die vernaamste bron van sydigheid van beta-koeffisiënte op die JEB aan 'dun' handel toegeskryf word. In hierdie artikel word die toepasbaarheid van 'n skattingsprosedure wat vir die effek van 'dun' handel korrigeer, ondersoek. Die ondersoek dui aan dat die korrigeringsprosedure wesenlike verbeterings in die skatting van beta-koeffisiënte lewer. Verder word op grond van 'n empiriese ondersoek aanbevelings gemaak betreffende die parameters wat in die skatter vir die betas ingesluit moet word.

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The beta coefficient has long been one of the most important financial statistics in the investment community. Consequently this statistic has occupied a central role in the canon of established literature on Finance. Although Dimson (1979) has asserted that the thinly traded phenomenon may result in beta estimation problems on smaller markets, relatively little research has been conducted in this area as the majority of researchers have concentrated on the large, efficiently functioning markets.

Bradfield (1989) shows that the extent of thin trading on the JSE is indeed significant and asserts that approximately one third of JSE stocks are not traded, on average, for at least one week out of every four-week period. Furthermore Bradfield (1989) demonstrates that the thinly traded phenomenon is responsible for the major source of bias associated with the estimation of beta coefficients on the JSE. In this paper the suitability of a beta correction procedure proposed by Cohen, Hawawini, Maier, Schwartz & Whitcomb (1983) (for use in thinly traded environments) is examined for JSE stocks.

Introduction

The fundamental concept of systematic risk, or beta, of a security is central to Capital Market Theory and consequently much empirical work has focused on the associated estimation problems. One of the more fruitful areas of empirical research arose from seeking the source of the estimation problems. Fisher (1966) was one of the first to identify the phenomenon of thin trading, but it was only relatively recently that Ball (1977) researched the effects of thin trading on the estimation of systematic risk. Scholes & Williams (1977) and Dimson (1979) were among the first to offer serious solutions to the estimation problems by developing a plausible analytical framework for the thinly traded phenomenon.

Several other approaches to the problem have, however, been suggested. For example, Ibbotson (1975), Dimson (1979) and Schwert (1977) introduced lagged market returns as additional independent variables in their market model regressions. Marsh (1979) and Franks, Broyles & Hecht (1977) on the other hand, used returns calculated only over periods when trades occurred, and regressed these returns on the market index over precisely the same periods. Neither of the above approaches takes into account the fact that in thinly traded markets the market index itself suffers from the effect of having component securities that are infrequently traded.

Scholes & Williams (1977) combined these ideas by using both non-synchronous and synchronous market returns as explanatory variables for trade-to-trade returns. Dimson (1979), however, points out that this method suffers from the disadvantage of requiring transaction dates, and also fails to make use of share prices which are not preceded, or followed, by a trade in an immediately adjacent time period. Dimson (1979) therefore, proposed a similar approach that largely overcomes these drawbacks. This approach does not require all components of the market index to be continuously traded, nor does it require information on the transaction dates. Cohen *et al.* (1983) later identified some inconsistencies in Dimson's derivation, and therefore proposed a modified version of Dimson's estimator. The estimator proposed by Cohen *et al.* (1983) will therefore be used in this investigation.

Theoretical development

The main cause of the bias associated with estimation problems in thinly traded environments is the fact that *recorded* prices are used to represent true *underlying* prices. For example, when a security has not been traded in the period in question then the *recorded* price of the security remains unchanged, and represents the outcome of a transaction in some previous period. The *underlying* (theoretical) price of the security, by contrast, would reflect the arrival of any new information in the period in question. Hence two series of prices are created, a series of *recorded* prices and an unknown, and more volatile, series of *underlying* prices. Clearly estimation problems arise when the series of *recorded* rather than *underlying* prices are used in the estimation procedures. In particular it is evident that the covariance between *recorded* security returns and the market's return, is likely to be less than the covariance between *underlying* security returns and the market's return for thinly-traded securities. This is clearly due to the fact that underlying prices reflect movements in the market instantaneously while recorded prices may remain unchanged. Since the OLS beta estimate embodies this covariance component in the numerator, it is evident that OLS estimates of beta for thinly traded will be underestimated when recorded prices are used in the estimation process. Even beta estimates for well-traded shares in a thinly traded environment may be subjected to estimation bias. This occurs because in thinly traded markets, the market index itself may be comprised of a significant proportion of thinly traded securities. This implies once more that an *observed* series and a *underlying* series of market index returns exist as well, and that this may cause further estimation problems, even for well-traded securities.

The beta estimator proposed by Dimson (1979) and corrected by Cohen *et al.* (1983) is designed to overcome the above problems by incorporating both lags and leads of the relevant return series in the analytical framework. Furthermore, Cohen *et al.* (1983) argued that the estimator has general applicability as it converges to the usual OLS beta estimate in well-traded markets. This estimator can be written as:

$$\hat{\beta}_j = \frac{b_j^0 + \Sigma_{n=1}^{N} b_{j+n}^0 + \Sigma_{n=1}^{N} b_{j-n}^0}{1 + \Sigma_{n=1}^{N} b_{M+n}^0 \Sigma_{n=1}^{N} b_{M-n}^0} \qquad (1)$$

where the ^{0} superscripts denotes the coefficient has been estimated from a series of observed, or recorded prices; and N denotes the number of leads, or lags used.

Furthermore estimates of the right-hand side components of the above expression can be obtained as follows:

$$b_j^0 = \frac{\text{cov}(R_{j,t}^0; R_{M,t}^0)}{\text{var}(R_{M,t}^0)}$$

$$b_{M+n}^0 = \frac{\text{cov}(R_{M,t+n}^0; R_{M,t}^0)}{\text{var}(R_{M,t}^0)}$$

$$b_{M-n} = \frac{\text{cov}(R_{M,t-n}^0; R_{M,t}^0)}{\text{var}(R_{M,t}^0)}$$

$$b_{j+n}^0 = \frac{\text{cov}(R_{j,t+n}^0; R_{M,t}^0)}{\text{var}(R_{M,t}^0)}$$

$$b_{j-n}^0 = \frac{\text{cov}(R_{j,t-n}^0; R_{M,t}^0)}{\text{var}(R_{M,t}^0)}$$

where $R_{M,t}^0$ is the observed return on the market index at time t; and $R_{j,t}^0$ is the observed return on security j at time t.

In order to justify the use of the proposed estimator, it is worth considering the structure of (1) in more detail. Firstly it is evident that in perfectly efficient, and hence well-traded markets, market efficiency ensures that all non-synchronous covariances will in theory be equal to zero. This implies that in (1) $\hat{\beta}_j = b_j^0$, i.e. the estimator, converges to the usual OLS beta in efficient markets. Other than the term, b_j^0, the components in the numerator of (1) capture the relationship between leads and lags of the security and the contemporaneous market index. The components of the denominator on the other hand, reflect adjustments for autocorrelations induced into the market index by the component thinly traded securities.

Suitable beta estimation on the JSE — an empirical investigation

The data

In order to investigate the suitability of the Cohen estimator on the JSE, the entire population of shares (amounting to 671) currently recorded on the JSE data tape was considered for selection over the period 1 January 1978 to 31 August 1987. With the view of simplifying computations only shares listed over this entire period were selected. This amounted to 360 shares in total. These 360 shares were subsequently ranked according to various criteria relating to their trading frequency. A share having a weekly volume of zero was taken to indicate that the share was not traded during a particular week.

Furthermore, two market indices were used in this investigation, the JSE-Actuaries Overall Index (a market capitalization type index), and an equally weighted index constructed to be the equally weighted average price of the 360 securities in each week. Finally, series of returns were constructed for each security and both market indices for use in the analysis.

Methodology

The securities were ranked according to the number of weeks that no trades[1] occurred for each security over the sampled period. The securities were then partitioned into 10 deciles on this basis, each consisting of 36 securities.

The methodology proceeds along the same lines as that of Dimson (1979), with the exception that the final beta estimator used here was the one proposed by Cohen *et al.* (1983). In essence, the only major difference between the two estimators is the fact that the non-synchronous coefficients of the Dimson estimator were estimated from a single multivariate regression model, while Cohen *et al.* used separate bivariate regression models to estimate each non-synchronous coefficient.

To obtain the component beta coefficients of equation (1), the weekly returns for each of the 360 securities were regressed against lagged, leading, and matching market index returns. This procedure was repeated using both the JSE-Actuaries Overall Index and the Equally Weighted Index. To investigate which, if any, of the lags and leads are significant for JSE stocks, five lags and five leads were considered in the investigation.

Results using the JSE-Actuaries Overall Index

Table 1 shows the resulting component beta coefficients, averaged over each decile at the various lags and leads using the JSE-Actuaries Overall Index as the market index.

The most notable feature of Table 1 is the fact that the component beta coefficients are largest for the synchronous data, i.e. having a lag of zero, as expected, with the exception of the 8th, 9th, and 10th deciles representing the most thinly traded securities having unexpectedly small synchronous coefficients, with decile 10 having an average coefficient of only 0,05. Furthermore the average beta coefficients for the synchronous data, decrease monotonically from 1,30 for decile 1 to 0,05 for decile 10. Note that these coefficients computed at the lag zero represent the usual, unadjusted beta coefficients. Since there is no plausible reason why the average beta coefficients should decrease systematically to this extent, it is clear that a severe bias due to thin trading is evident for JSE stocks. The last column of Table 1 shows the corrected beta estimator proposed by Cohen *et al.* (1983). It is clear that the proposed estimator does improve beta estimation for the thinly traded deciles, however there appears to be an overestimation bias for the well-traded securities. In particular, decile 1 has an average beta estimate of 2,62, and this value appears to be too large within this framework to be economically plausible. Nevertheless, in order to determine which leads and lags should ideally be used with the JSE-Actuaries Overall Index for beta estimation, the associated t statistics for the component beta coefficients were averaged for each decile at the various leads and lags. Table 2 shows the resulting average t statistics for the component beta coefficient obtained using the JSE-Actuaries Overall Index.

Table 1 Average component beta coefficients for the JSE-Actuaries Overall Index

	Lag or lead											Cohen estimator
Decile	-5	-4	-3	-2	-1	0	1	2	3	4	5	
1	0,11	0,07	0,05	0,18	0,17	1,30	0,26	0,19	0,05	0,09	0,16	2,62
2	0,12	0,08	0,04	0,11	0,23	0,92	0,17	0,10	0,00	0,06	0,07	1,91
3	0,07	0,05	0,04	0,13	0,30	0,74	0,15	0,06	0,07	0,04	0,03	1,70
4	0,12	0,09	0,07	0,16	0,33	0,60	0,09	0,06	0,01	0,02	0,01	1,56
5	0,11	0,05	0,06	0,18	0,31	0,45	0,04	-0,01	0,01	0,01	0,05	1,26
6	0,13	0,09	0,09	0,19	0,35	0,44	0,07	0,00	0,00	0,03	-0,01	1,38
7	0,11	0,11	0,12	0,18	0,27	0,29	0,05	-0,00	0,03	0,00	-0,02	1,14
8	0,11	0,08	0,16	0,11	0,24	0,19	0,13	0,02	0,08	0,00	-0,01	1,12
9	0,13	0,12	0,09	0,15	0,15	0,11	-0,01	-0,01	-0,02	0,00	-0,02	0,68
10	0,05	0,04	0,06	0,06	0,06	0,05	-0,01	-0,01	0,03	0,02	-0,01	0,34

Decile 1 consists of the most frequently traded securities whilst decile 10 consists of the most infrequently traded securities

Table 2 average t statistics of the component beta coefficients for the JSE-Actuaries Overall Index

	Lag or lead										
Decile	-5	-4	-3	-2	-1	0	1	2	3	4	5
1	1,05	0,76	0,49	1,81	1,86	[18,84]	[2,70]	1,92	0,47	0,93	1,60
2	1,24	0,80	0,39	1,21	[2,58]	[12,64]	1,93	1,15	0,03	0,59	0,67
3	1,02	0,63	0,68	1,52	[3,59]	[9,18]	1,73	0,75	0,43	0,45	0,26
4	1,21	0,87	0,77	1,75	[3,86]	[6,72]	1,09	0,52	0,04	0,26	0,09
5	1,48	0,72	0,65	1,79	[3,39]	[4,46]	0,67	-0,18	0,17	0,20	0,33
6	1,28	0,76	0,91	[2,01]	[3,31]	[3,66]	0,36	-0,02	0,25	0,27	-0,15
7	1,18	1,17	1,23	[2,02]	[3,05]	[2,88]	0,42	0,05	0,288	0,02	-0,22
8	1,25	1,14	1,33	1,81	[2,80]	[2,14]	0,48	0,15	0,19	0,072	-0,07
9	1,56	1,48	1,08	1,85	1,83	1,30	0,26	-0,07	-0,35	-0,01	-0,22
10	0,58	0,42	0,63	0,67	0,64	0,51	-0,06	-0,15	0,44	0,18	-0,10

The deciles having significant average t statistics (at the 5% level) for the component beta coefficients are boxed in

Inspection of the resulting average t statistics in Table 2 reveal results similar in spirit to those found in Dimson (1979). It is evident that all synchronous coefficients (i.e. at lag zero) are significant with the exception of deciles 9 and 10, representing the extreme thinly traded[2] cases. Furthermore the coefficients at 1 lag are found to be generally significant with the exception of decile 1, and decile 9 and 10. A further notable feature is the fact that several of the leading coefficients for the thinly traded shares (deciles 6 to 10) are negative.

These results are consistent with the intuitive arguments given by Dimson (1979). The major intuition behind Dimson's arguments are that for frequently traded securities the leading coefficients are more important as they 'lead' a market index suffering from thin-trading effects. Whilst for infrequently traded securities the lagged coefficients are more important as they generally lag behind the market index.

From the above analysis it is claimed that the inclusion of one lagged and one matching coefficient in equation (1) would appear to be sufficient as a general rule for estimation of beta on the JSE[3] using the JSE-Actuaries Overall Index. Although it was found that the final beta estimator was still not ideal, it is felt that this may be due to the fact that the JSE-Actuaries Overall Index (a market capitalization type index), rather than the estimator (1), is not suitable for use in beta estimation procedures. The analysis was thus repeated using the constructed Equally Weighted Index in the estimation procedure.

Results using the Equally Weighted Index

The major distinction between the two indices used here is that the JSE-Actuaries Overall Index is comprised of almost exclusively well-traded securities having relatively large market capitalization proportions. The Equally Weighted Index on the other hand includes all thinly traded securities as well, all being given the same weight. Consequently the Equally Weighted Index itself is likely to suffer from the effects of thin trading to a far greater extent than the JSE-Actuaries Overall Index. Furthermore, the usual beta coefficients of the component securities of an equally weighted index will necessarily average out to unity, when the equally weighted index is used as the independent variable. In thinly traded environments this averaging suggests that betas of well-traded securities are usually overestimated, while those of infrequently traded securities are underestimated.

Table 3 shows the resulting component beta coefficients averaged over each decile at the various lags and leads using the Equally Weighted Index as the market index. From Table 3 it can be seen that the synchronous beta coefficients (at lag zero) again decrease monotonically, from 1,74 for decile 1 to 0,21 for lag 10, again indicating that the beta coefficients are overestimated for well-traded securities and underestimated for thinly traded securities, as expected. The final beta estimators shown in the last column of Table 3 range from 1,52 to 0,43 compared to the equivalent column of Table 1 which range from as high as 2,62 to 0,34.

The final average beta estimators using the Equally Weighted Index thus appear to be more economically plausible than those of Table 1. Furthermore, for the Equally Weighted Index the final beta estimator is seen to make corrections which are consistent with the theoretical intuition. For example, for decile 1, representing the well-traded securities, the coefficient at lag zero, i.e. 1,74 was identified as an overestimate of beta, the final estimate, i.e. 1,52 is seen to be corrected downward, i.e. in the right direction. By contrast the correction for decile 1 in Table 1 is seen to be counter-intuitive. The corrections for the other deciles in Table 3 also appear to be consistent with intuition, resulting in final estimations fairly close to one. The final estimators still, however, appear to decrease slightly down the deciles in Table 3, although this decrease is not as extreme as the case shown in Table 1.

Table 3 Average component beta coefficients for the Equally Weighted Index

	Lag or lead											
Decile	-5	-4	-3	-2	-1	0	1	2	3	4	5	Cohen estimator
1	0,07	0,07	0,09	0,16	0,25	1,74	0,83	0,49	0,21	0,24	0,41	1,52
2	0,13	0,09	0,09	0,09	0,33	1,44	0,70	0,36	0,16	0,17	0,25	1,27
3	0,11	0,06	0,04	0,14	0,45	1,34	0,63	0,33	0,24	0,20	0,20	1,25
4	0,14	0,11	0,10	0,24	0,58	1,19	0,54	0,27	0,16	0,14	0,15	1,21
5	0,18	0,11	0,08	0,29	0,60	1,11	0,48	0,19	0,13	0,11	0,15	1,14
6	0,20	0,14	0,14	0,29	0,67	1,07	0,45	0,21	0,10	0,13	0,13	1,18
7	0,19	0,19	0,25	0,35	0,58	0,77	0,36	0,18	0,14	0,11	0,07	1,06
8	0,19	0,17	0,29	0,31	0,59	0,75	0,37	0,24	0,25	0,09	0,14	1,10
9	0,27	0,27	0,27	0,34	0,41	0,39	0,15	0,10	0,05	0,07	0,05	0,80
10	0,13	0,12	0,19	0,17	0,20	0,21	0,06	0,01	0,08	0,07	0,04	0,43

Table 4 Average t statistics of the component beta coefficients for the Equally Weighted Index

	Lag or lead										
Decile	-5	-4	-3	-2	-1	0	1	2	3	4	5
1	0,42	0,46	0,51	0,99	1,65	[13,51]	[5,64]	[3,18]	1,39	1,52	[2,61]
2	0,90	0,56	0,45	0,55	[2,26]	[11,19]	[4,89]	[2,49]	1,12	1,09	1,60
3	0,82	0,39	0,48	1,18	[3,37]	[9,66]	[4,33]	[2,20]	1,37	0,27	1,23
4	0,96	0,71	0,66	1,61	[4,14]	[0,49]	[3,77]	1,79	1,07	0,96	0,92
5	1,15	0,83	0,62	1,81	[3,95]	[6,75]	[3,05]	1,15	0,94	0,77	0,93
6	1,25	0,69	0,86	[1,97]	[4,06]	[5,91]	[2,33]	1,12	0,81	0,83	0,61
7	1,31	1,29	1,59	[2,41]	[3,98]	[4,86]	[2,18]	1,17	0,92	0,80	0,47
8	1,54	1,43	1,83	[2,42]	[0,83]	[4,17]	[1,99]	1,09	0,89	0,56	0,56
9	[2,12]	[2,07]	[2,08]	[2,73]	[3,23]	[3,07]	1,08	0,83	0,30	0,51	0,28
10	1,00	0,88	1,20	1,09	1,33	1,32	0,32	0,03	0,47	0,47	0,19

The deciles having significant average t statistics (at the 5% level) for the component beta coefficients are boxed in

Further support for using the Equally Weighted Index with the Cohen estimator can be seen by considering the associated average t statistics for the component beta coefficients obtained using the Equally Weighted Index. These results are shown in Table 4 and are similarly seen to be consistent with the theoretical preamble. Here at least one lagged and one leading coefficient, together with the matching coefficient, appear to be generally applicable for JSE stocks. For deciles 1, 2 and 3, representing well-traded securities, two leading coefficients are significant, while for deciles 6, 7, 8 and 9, representing the thinly-traded securities, two lagged coefficients are significant (several more are also significant for decile 9). The results for decile 10 in Table 4 show, by contrast, that none of the component beta coefficients are significant. As mentioned before the incidence of thin trading is likely to be so extreme for decile 10 that even five lags are probably insufficient to capture the desired effects.

These results were intuitively expected, as the well-traded securities are expected to 'lead' an equally weighted market index. This is a consequence of the fact that an equally weighted index has a positive autocorrelation induced by its component thinly traded securities. The thinly traded securities on the other hand, are themselves expected to 'lag' the market index.

In order to determine the effect of the thinly traded phenomenon on the two indices used, the denominator of equation (1) will be considered here. The denominator attempts to capture the extent to which the components of the indices induce a thinly traded component into the index itself, and is used to correct for this in model (1). For the five leads and five lags, the JSE-Actuaries Overall Index yielded a value of 1,966 for the denominator of equation (1). By contrast, the equally weighted index yielded a value of as high as 3,280 for the denominator of (1). Clearly the Equally Weighted Index is seen to reflect the significant degree of thin trading, induced by its component securities on the JSE, to a far greater extent than the JSE-Actuaries Overall Index.

Although the Equally Weighted Index itself does not escape the problem of thin trading, it does appear to yield more intuitively appealing estimates of beta (when used in conjunction with Cohen's estimator) than does the JSE-Actuaries Overall Index.

Conclusion

It can be concluded that the effect of thin trading on the estimation of beta coefficients is substantial on the JSE.

The estimator proposed by Cohen *et al.* (1983) was found to yield substantial improvements in the estimation of beta coefficients on the JSE. However, it was found that more satisfactory improvements can be achieved when an equally weighted index is used in conjunction with the beta estimator proposed by Cohen *et al.* (1983). Finally it was found that if the JSE-Actuaries Overall Index is used, at least one lagged and the matching coefficient should be included in the estimator. On the other hand, if an equally weighted index is used, as is recommended, at least one lagged, one leading, and the matching coefficient should be included in the estimator.

The results of this section show that there is possible scope for further improvement in the beta estimation procedure. Research in this direction is currently ongoing.

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Notes

1. This criterion was chosen as it gave the largest spread of trading frequencies across the deciles.
2. The thin trading in deciles 9 and 10 is so extreme that even six lags may be insufficient to capture these effects.
3. Dimson (1979) conducted a similar analysis for UK stocks using daily data; he found that at least four daily lags and one lead should be included with a matching coefficient.

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