Certainty equivalent coefficients and capital budgeting: a caveat

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The purpose of this technical note is to draw attention to the problems which are inherent in the use of certainty equivalent coefficients as an approach to incorporating risk into capital budgeting. More specifically, the certainty equivalent coefficient net present value criterion violates an important principle of cash flow determination for discounted cash flow analysis. Further, this approach precludes the use of net present value profiles which are pivotal when evaluating conflicts among mutually exclusive projects. In addition, use of certainty coefficient equivalents amounts to an acknowledgement that the concept, function and use of the cost of capital is improperly understood.

Die doel van hierdie tegniese verslag is om aandag te vestig op die probleme wat inherent deel is van die gebruik van sekerheidsekwivalent-koëffisiënte as 'n benadering tot die inlywing van risiko in die kapitaalbegroting. Die sekerheidsekwivalent-koëffisiënt netto teenswoordige waardekriterium oortree 'n belangrike beginsel van kontantvloeibepaling vir verdiskonteerde kontantvloei-ontleding. Verder sluit hierdie benadering die gebruik van netto teenswoordige profiele, wat van kardinale belang is in die oplossing van botsings onder onderling-eksklusiewe projekte, uit. Laastens dui die gebruik van sekerheidsekwivalent-koëffisiënte op 'n erkenning dat die konsep, funksie en gebruik van die vereiste opbrengskoers, nie heeltemal begryp word nie.

Introduction

The literature of capital budgeting identifies a number of ways in which risk can be accommodated within the valuation process (Clark, Hindelang & Pritchard, 1984: 159–193). As an approach, certainty equivalent coefficients enjoy a prominent literary position despite the fact that the computation of the certainty equivalent coefficient net present value is subject to a number of shortcomings. Firstly, it violates a fundamental principle of cash flow determination for purposes of discounted cash flow analysis. Secondly, it disenables the construction and use of net present value profiles and therefore precludes the application of Fisherian analysis for resolving problems related to the ranking of alternative projects. Thirdly, it amounts to an acknowledgement that the concept of the discount rate, the cost of capital, is improperly appreciated.

The purpose of this technical note is to draw attention to the above enunciated deficiencies of the certainty equivalent coefficient approach when calculating net present value.

Certainty equivalent coefficient approach

The certainty equivalent coefficient approach to the calculation of net present value is defined (Clark et al., 1984: 177):

$$\text{CEC(NPV)} = \sum_{t=1}^{n} \alpha_t A_t \div (1 + R_f)^t - I_o,$$

where:

CEC(NPV) = certainty equivalent coefficient net present value;

t = a time index which varies from one to n;

α_t = certainty equivalent coefficient in period t;

A_t = annual expected after-tax cash flow in period t (which may have either positive or negative values);

R_f = risk free rate of return which functions as the cost of capital;

n = the project's expected life; and,

I_o = initial outlay.

The certainty equivalent coefficient varies from one, which describes a situation of no risk, to zero, which describes a situation of complete uncertainty. Since risk is fully accommodated in the numerator of the above definition by the certainty equivalent coefficient, the cash flows are discounted at the risk free rate of return. Once it is acknowledged that this approach is a variant of net present value, it is not unreasonable to expect that this variant should not be in conflict with the fundamental principles used to determine cash flow for discounted cash flow analysis. Such conflict, which introduces inconsistency in the valuation process, is presented and discussed.

Principles of cash flow determination

For purposes of discounted cash flow analysis and hence capital budgeting, cash flow determination subscribes to a number of fundamental principles such as (Paulo, 1992: 179–180):

— only incremental revenues and incremental costs are relevant to the determination of the cash flow, consequently average, fixed, sunk, historic, pro-rata as well as overhead costs and revenues are ignored;

— finance charges do not feature in the computation of net cash flow since they are taken into account in the discount rate;

— working capital which is needed to sustain the optimal level of functioning of fixed assets, features as an outflow in the period in which it occurs, and features as an inflow at the end of the project's life; and

— depreciation, which constitutes a legitimate expense in the determination of taxable income, is subtracted from the cash flow, tax is then subtracted from the cash flow, and then depreciation is added back to the cash flow because cash has not flowed out of the project.

Before considering the impact of not adhering to the second principle enunciated above, namely that finance charges do not feature in the computation of net cash flows because they are taken into account in the discount rate, a brief review of the risk free rate of return is presented because of its pivotal role in the certainty equivalent coefficient criterion.

Risk free rate of interest (I)

The cost of capital comprises two major and distinct components, *firstly*, a risk free rate of return, and *secondly*, components which take into account the diversity of risks such as business, financial, inflation, term structure, expectations, and tax risk. In other words, the second group of components are characterized by a variety of probabilities attaching to a variety of outcomes, whereas the first group is *not* described by means of a probability distribution function. Moreover, the risk free rate of interest is common to all investors, so, the differences in the cost of capital must originate in the second group of components, the risk premia components.

The impact of monetary instability has long been recognized in the financial literature and its effect in determining the nominal interest rate was postulated in 1896 by Irving Fisher (Van Horne, 1986: 148, 565–566). This postulation, known as the Fisher Effect, expresses the nominal rate of interest on a security as the sum of the real interest rate, the rate of price change expected over the maturity of the security and the cross product, and is algebraically revealed by the Fisher Equation (Copeland & Weston, 1988: 61):

$$[\ (1+R_f) = (1+i)(1+h)\ ],$$

where:
R_f = nominal risk free rate of interest;
i = real risk free interest rate;
h = expected or anticipated inflation rate.

In terms of the Fisher Effect, the purchaser of a security requires a nominal rate of interest sufficiently high so that a real rate of interest can be earned.

The determination of the risk free real rate of interest is thus a composite of two factors, the time preference of consumption, and, opportunity cost. The time preference of consumption implies that savers will postpone present consumption only when adequately compensated, and this is influenced by the opportunity cost of parting with financial resources. When funds are invested, the investor foregoes the opportunity of using those funds for current consumption, and, even if he is certain that he will recover those funds in the future, compensation is demanded for sacrificing current consumption. The consumption takes the form of the interest rate available on a riskless investment such as United States treasury bills, which are free from the risk of default (Brigham, 1985: 68). Thus, this compensation only covers the temporary sacrifice of abstinence from consumption.

The risk free rate is also influenced by investors' expectations about future rates of inflation because they will seek to protect themselves from declines in future consumption possibilities. Abstinence from present consumption does not dictate lower levels of future consumption, so, when inflationary expectations increase, investors demand a higher return on risk free investments. Thus, the risk free rate of interest is a risk free real rate of interest (Brigham, 1985: 66–67).

Extensive tests of this proposition, the Fisher Effect, (Hendershott & Van Horne, 1973: 301–314; Carghill & Meyer, 1974: 458–471) have revealed that real default free interest rates were relatively stable in the United States of America during the 1950s and 1960s; since the 1970s though, expected real default free interest rates have fluctuated (Fama, 1975: 269–282; Nelson & Schwert, 1977: 478–486; Hess & Bicksler, 1975: 341–360). The impact of these fluctuations has been relatively minor on the real default free interest rate, but nominal interest rates have fluctuated in keeping with anticipated inflation, thus vindicating the thesis of the Fisher Effect as embodied in the Fisher Equation (Ben-Horim, 1987: 234; Clark, Hindelang & Pritchard, 1984: 6). In other words, the risk free rate of return, which is a real rate of return, is subject to minimal variability, and embodies an element attributable to expected future inflation.

Providing the inflation rate is expected and is anticipated correctly, it is not a source of risk; however, when it is not anticipated, then it is a source of risk and this aspect must be brought into account as an inflation risk premium (Van Horne, 1986: 81; Mittra & Gassen, 1981: 123) in addition to the fully anticipated risk premium within the risk free rate.

The significance of the concept of the risk free rate of interest in financial management is patent when the following illustrations are considered. *Firstly*, one of the assumptions of *portfolio theory* is the existence of a risk free asset yielding a risk free rate of return within a perfect capital market (Bromwich, 1977: 313). The risk free rate of return, proxied by the rate of interest on treasury bills, in the case of the United States, forms an integral part of the *capital asset pricing model* approach to the cost of capital. According to this model, the cost of capital may be determined (Mittra & Gassen, 1981: 545):

$$R_j = R_f + B_j\ [\ E(R_m) - R_f\ ]$$

where:
R_j = cost of capital of asset j;
R_f = risk free rate of return;
$E(R_m)$ = the expected return on the market index; and
B_j = beta coefficient of asset j.

Secondly, in capital budgeting, the risk free rate of interest is central to the *certainty equivalent coefficient* approach. Whereas the net present value approach combines the discounting for time together with the adjustment for risk, and *it is the risks which primarily give rise to costs*, the certainty equivalent coefficient approach disaggregates these two factors by adjusting for risk with a certainty equivalent coefficient, and discounting for the time value of money at the risk free rate of interest.

Thirdly, the risk free rate of interest serving the function of the required rate of return can be used to calculate net present value when conducting *simulation analysis* (Lewellen & Long, 1972: 19–32), within a simulation model such as the Monte Carlo model (Hertz, 1964: 96–108).

Thus, the risk free rate of interest forms an integral part of the cost of capital, and under certain circumstances, such as in the case of the certainty equivalent coefficient approach, may be regarded as the cost of capital, the discount rate.

Finance charges: a discount rate item or a cash flow item?

It is important to reiterate that finance charges, which include the explicit and implicit costs as well as the risks of finance capital, do not feature in the computation of cash flow because they form part of the discount rate (the cost of capital). Net present value is calculated according to the assumption that this fundamental principle is operative. If net present value is modified for purposes of risk, then how legitimate can a modification, which is a variant of net present value, possibly be if it entails a violation of a fundamental determinant of net present value?

If finance charges are no longer included in their entirety as part of a percentage in the discount rate (the cost of capital), they then have to be converted to a monetary amount and included in the cash flow, and then somehow their associated risk has to be incorporated in the certainty equivalent coefficient. In other words cost has to be disaggregated from risk. However, as already argued, it is risks which give rise to costs (and also to returns as revealed by the beta coefficient of the capital asset pricing model). Where debt finance is used, the term structure of interest rates needs to be incorporated into the numerator of the certainty equivalent coefficient net present value. From the definition of the term structure of interest rates, namely the relationship between yield and maturity on debt securities which differ only in the length of time to maturity (Mittra & Gassen, 1981: 204), it is evident that both costs and risks form a composite percentage within the three main theories which have been proposed to explain the term structure of interest rates, namely the unbiased expectations theory (Copeland & Weston, 1988: 66; Lutz, 1940: 36–63), the uncertainty and liquidity premia theory (Keynes, 1936: 168, 182, 201; Hicks, 1946: 164; Polakoff & Durkin, 1981: 519), and the market segmentation theory (Walker, 1954: 22–23; Culbertson, 1957: 489–504).

The disaggregation of cost from risk so that risk can be incorporated into the certainty equivalent coefficient and cost can be included in the cash flow, could prove to be a somewhat challenging exercise. Even if such adjustment were correctly performed, such a procedure nonetheless introduces inconsistency into the valuation process, and moreover disenables the comparison and therefore ranking of alternative projects according to the criterion of net present value.

Net present value profiles and ranking

Net present value profiles, which graphically portray the change in net present value as the cost of capital changes, are a useful form of sensitivity analysis. In the event of conflicts in the ranking of projects, net present value profiles are constructed, superimposed on each other, and a Fisherian analysis (Clark et al., 1984: 65–66) based on the interactive relationship of the cost of capital with the internal rate of return is performed.

If a certainty equivalent coefficient net present value approach is adopted, it is not possible to construct net present value profiles because according to this criterion, the cost of capital is the risk free rate. The risk free rate, which comprises time preference and fully anticipated inflation only (Block & Hirt, 1987: 279–280; Fisher, 1954: 61–94, 141–143, 181–183), has been shown to be relatively stable over time (Fama, 1975: 269–282; Nelson & Schwert, 1977: 478–486). Providing the inflation rate is expected and is anticipated correctly it is not a source of risk; however, when it is not anticipated, then it is a source of risk and is brought into account as an inflation risk premium (Van Horne, 1986: 81; Mittra & Gassen, 1981: 123), and does not feature in the risk free rate.

If the risk free rate, which is a relatively stable quantum, functions as the cost of capital, then changes in net present value cannot be attributed to the risk free rate. Consequently, it is not possible to construct net present value profiles, for the question arises as to what is being measured on the horizontal axis. The risk free rate does not vary from zero to some relatively large value. In the absence of net present value profiles, conflicts in the ranking of projects cannot be resolved using a Fisherian analysis, and in the currently available literature on capital budgeting no alternative to a Fisherian analysis has been reported.

In brief, use of a certainty equivalent coefficient approach precludes the use of net present value profiles, thereby severely restricting financial analysis.

Cost of capital as a concept

The cost of capital, also known as the required rate of return, is defined as the minimum rate of return necessary to maintain investor wealth intact. As such, it neither augments nor depletes investor wealth, and therefore any factor which could jeopardize investor wealth is accommodated in the cost of capital as a risk premium. As a concept, the cost of capital comprises two main components, the risk free rate and risk premia. The risk free rate is common to all investors. The differences which describe investors as individuals and the markets within which they transact, are important sources for the identification of investor-specific risks. These risks are accommodated in the risk premia component of the cost of capital. The function of the cost of capital is to provide guidance in the acquisition and allocation of financial resources. This concept is used to evaluate investment, financing, and dividend decisions, although for purposes of this article attention is focused on long term investments, namely capital budgeting.

When the cost of capital is correctly specified, all factors which could jeopardize investor wealth are specifically taken into account as risk premia, and then there is no need to make use of certainty equivalent coefficients. The existence and discussion of certainty coefficient equivalents, as an approach to risk in capital budgeting, constitutes *prima facie* evidence that an incomplete understanding of the cost of capital is prevalent, in which case the cost of capital is either misspecified, austere, or both. This situation prompts the question as to what is being used as the basis for the determination of the cost of capital.

Summary and conclusion

In this article the certainty equivalent coefficient net present value approach was critically examined and found to be deficient in three regards. *Firstly*, as a variant of net present value it violates an important fundamental principle which is integral to the determination of cash flow for discounted cash flow analysis, and hence important to the net present value criterion.

Secondly, use of the certainty equivalent coefficient net present value approach precludes the use of net present value profiles, a serious shortcoming particularly when conflicts in project rankings arise. *Thirdly*, the discussion and use of certainty coefficient equivalents as a means for incorporating risk reveals a misconception of the definition function and use of cost of capital. Provided that the cost of capital is correctly understood and specified, there should be no need to contemplate using a certainty equivalent coefficient approach for the cost of capital will correctly reflect the risk of the project being evaluated.

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