

Relationship between share index volatility, basis and open interest in futures contracts: The South African experience

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In a rational efficiently functioning market, the price of the share index and share index futures contracts should be perfectly contemporaneously correlated. However, in practice the cost of carry model is obscured as the basis varies and is normally not equal to the cost of carry.

This study uses the Chen, Cuny and Haugen (1995) model to examine the relationship between the basis and volatility of the underlying index and between the open interest of the futures contract and the volatility of the underlying index. The tests were performed on data from ALSI, FINI and INDI futures contracts. The sample period was from January 1998 to December 2001.

The results confirm the conclusion of Chen *et al.* (1995) who found the basis to be negatively related to the volatility of the underlying index. The other main prediction of the Chen *et al.* (1995) model, which is also supported by the current study, is that open interest is significantly related to the volatility of the underlying index. The results further support the proposition of Helmer and Longstaff (1991) of a highly significant negative concave relationship between the basis and the interest rate.

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Introduction

The cost of carry model is undoubtedly the most popular model for pricing share index futures (Berglund & Kabir, 1995). The model expresses the futures price in terms of the underlying share index value, the risk-free rate and the dividend yield for the index (Helmer & Longstaff, 1991). According to the model, at equilibrium the futures price should equal its fair value (the fair value is spot price less any dividends that accrue before the expiry date plus financing costs). At any given point in time, the futures price will be equal to the spot price plus the net financing cost, which is normally called the cost of carry (dividends earned on the asset until the settlement date less financing cost for borrowing and lending until settlement date). As delivery approaches, financing costs will approach zero and the dividend that can be earned by holding the share will also approach zero and consequently the two prices will converge.

Despite its popularity, there is a great deal of empirical evidence that shows that in practice the cost of carry model is obscured. Cox, Ingersoll and Ross (1981) showed that the futures and forward prices are different when interest rates are stochastic. Helmer (1988) suggested that price deviations from the cost of carry model are related to the level of interest rates or the volatility of the underlying security price in some financial futures markets. He also showed that technological changes induce randomness

which cause the futures and the spot prices to lose alignment. Helmer and Longstaff (1991) demonstrated that arbitrage models that assume that the share market to be exogenous, normally fail to capture the dynamic interactions between the spot and future markets.

The cost of carry model makes some assumptions that are not always true in practice. Whenever the assumptions are violated, the difference between the spot and the futures prices will not be equal to the basis. Harvey and Whaley (1991) showed that asset mispricing could result from the cost of carry assumption that the dividend yield is constant. A number of researchers have shown that more often than not futures prices predicted by the cost of carry model differ from the value observed in the market, the reasons being *inter alia* the tax treatment of futures and spot markets, and the existence of timing options available in spot and not in futures markets.

In an attempt to examine deviations of the share index futures prices from their cost of carry value, this study will apply the Chen, Cuny and Haugen (1995) model to data from the South African Futures Exchange (SAFEX). This will be accomplished by using three share index futures contracts namely the ALSI, FINI and the INDI.

The study examines how volatility affects the basis as well as the open interest of the share index futures. According to Chen *et al.* (1995), as volatility increases, investors will sell

futures in order to hedge their stock position. An increase in volatility means an increase in the risk of the stock. As a result, there will be an increase in the expected return of the share. As more investors come in to share in the risk of the share, the share price will go up relative to the futures price. As a result the basis will decrease.

Chen *et al.* (1995) also predicted that as volatility increases, open interest would follow. There is a positive relationship between volatility of the underlying asset and the open interest of the share index futures. As volatility increases, investors will sell more futures to hedge their share position. As short positions increase, some investors will enter to take the long side of these positions. As a result, open interest will rise.

The study will test two predictions of the CCH model, namely that:

- the basis decreases as the volatility of the index increases, and
- open interest of index futures increases as the volatility of the index increases, as well as

a proposition of Helmer and Longstaff (1991), which implies that the basis has a negative concave relationship with the interest rate.

The article is structured as follows. Section 2 sets out the cost of carry model, while Section 3 deals with the data and Section 4 with the empirical results. A conclusion follows.

The cost of carry model

The cost of carry model is a no-arbitrage pricing relationship that assumes that at maturity spot prices and futures prices must converge. The model expresses the futures prices in terms of the underlying share index value (spot price), the risk-free rate and the dividend yield. The futures price is:

$$F_t = S_t e^{(r-d)t} \quad \dots (1)$$

where:

F_t is the futures price at time t

S_t is the spot price at time t ;

t is the time left to maturity;

d is the dividend yield; and

r the risk-free rate.

The above model assumes that the spot asset provides a continuous dividend yield and recognises that an investor should be indifferent to:

- purchasing the asset and reinvesting the dividends and
- purchasing a futures contract and investing in a risk-free bond.

According to the model, the difference between the two prices at any given time is the cost of carry. The cost of carry equals the basis. As the maturity date approaches, days left to maturity (t) decrease and consequently the cost of carry will decline until it is zero at maturity.

According to Keynes (1930) and Hicks (1939), speculators will enter the market only if they expect a positive profit. If they are holding long positions, it implies that the futures price is less than the expected spot price. They expect to buy at lower future prices and sell at higher spot prices, thus locking in a risk-free profit.

In practice, the market is not perfect and consequently the assumptions stated above may be violated. This complicates and disturbs the relationship in equation 1 and, as a result, the difference between the futures price and the spot price, that is the basis, will not always be equal to the cost of carry. In fact, prices predicted by the cost of carry model are frequently significantly higher than those observed for the share index contracts. Several reasons for this have been identified:

- **Taxation and interest rate:** The model assumes zero taxation. However, in practice, capital gains are taxable. Capital gains tax came into effect in South Africa on 1 October 2001. Both the capital gains tax and income tax have an effect on the futures prices. An increase in the income tax rate reduces both the effective dividend yield and the effective interest rate. For example, a 10% increase in the income tax rate will reduce both the dividend yield and the interest rate by 10%. In cases where interest rates are higher than the dividend yield, the net effect will be a reduction in the futures price.

The effect of capital gains tax depends on the relationship between the spot and the futures prices. In cases where the futures price is higher than the spot price, traders who hold futures pay less tax than those who hold shares. Therefore, if futures prices are higher than the spot prices, capital gains tax increases the value of the futures and makes them attractive compared to holding shares.

- **Timing option:** Another important issue related to tax is the timing option. The cost of carry model ignores the effect of the timing option. Shareholders have a valuable timing option in the sense that they can reduce their tax by deferring their capital gains and realising capital losses. An investor who holds futures does not have this option. In futures contracts, all capital gains/losses are realised either at the end of the year or at maturity, whichever comes first. As a result of this, futures prices will be lower than predicted by the cost of carry model.

- **Transaction costs:** The model also assumes that there is no transaction cost. However, in practice, investors pay a variety of transaction costs. These include costs associated with entering and closing positions, ask-bid spread etc. and these costs affect the futures prices.

- **Borrowing and lending rate:** In a perfect market, investors can borrow and lend at the same rate. In practice, the borrowing and lending rates are not the same. Generally the borrowing rate is higher than the lending rate.
- **Dividends:** The model also assumes that dividends are known with certainty and that the timing of the receipt of dividends from each stock of the index is known with certainty. Furthermore, the model assumes that there are no interim cash flows such as interim dividends. In practice, the dividend amount and timing are predictable, but not with certainty. Amounts and payment dates can be predicted based on past policy of the company. However, these are far from certain until the company announces the amount and payment date of the dividends. The cost of carry model also assumes that the dividend flow from the underlying asset is constant. However, in practice, most firms pay dividends more than once a year and these dividends are usually not the same. These seasonal variations in dividends have an impact on the observed futures prices. A model that assumes that dividends are constant throughout the year misprices the futures contracts. A contract where smaller dividends are paid is overpriced relative to where higher dividends are paid.

In perfect markets, Equation 1 gives an exact equation for the relationship between the spot and the futures prices. Deviation from this no-arbitrage price will cause investors to reap a risk-free profit without investments. For a market with imperfections as discussed above, the relationship is modified to give an arbitrage range within which the futures price should be. If the futures prices go beyond these boundaries, arbitrage profits become possible.

Helmer and Longstaff (1991) gave a number of reasons why the fair value will always differ from the value predicted by the cost of carry model. One of their reasons was that the cost of carry model, which is based on the no-arbitrage principle, assumes that the market is exogenous and as such fails to capture the dynamic interactions between the spot and futures markets.

Cox, Ingersoll and Ross (1981) found that futures and forward prices differ in predictable ways. Their model predicts that the price for a forward contract relates to the interest rate on the long-term bond that matures at the same time as the contract, while the futures price is related to the return from rolling over one-day bonds until contract maturity. These prices will be identical only in cases where the interest rates are non-stochastic.

Stoll and Whaley (1990) reported variations from the cost of carry model as a result of transaction costs. In their study, they used hourly S&P 500 Index and index futures data for the period April 1982 to December 1985. Their results showed that the cost of carry relation was violated in 80% of June 1982 contracts. However, the frequency fell below 15% for more recent contract maturities. MacKinlay and Ramaswamy (1988) reported similar results for S&P 500 futures contracts. Using 15-minute price data, they found

that the cost of carry relation was violated 14,4% of the time on average. Their data indicated that futures price changes are uncorrelated and that the variability of these price changes exceeds the variability of price changes in the S&P Index.

The other important reason is that the two markets differ in sensitivity. This means that new information has different effects on the two markets. The futures market is believed to be more sensitive to new information than the spot market. For example, if the news event causes the spot price to change by 2%, the futures price will change by more than 2%. As a result the value predicted by the cost of carry model will be lower than the observed futures price (Personal opinion, Futures market economist).

Sim and Zurbreugg (1999) maintain that a departure from the cost of carry relation is due to the fact that there are fewer trading constraints in the futures markets than in the spot market. This renders the futures market more information efficient than the underlying index market. According to Sim and Zurbreugg (1999), the other contributing factor is the liquidity and lower trading costs of the futures market as compared to the stock market. This is in agreement with the findings of Stoll and Whaley (1990) who examined intraday price changes from S&P and MM share index and futures contracts for a serial correlation using an ARMA process. They also believe that lower trading costs and liquidity in futures markets play a vital role in explaining the systematic evidence that futures markets often lead stock markets. 'There is evidence that the futures market leads the stock market and this is attributable only in part to the fact that not all stocks in the index trade continuously. The remaining predictive power of the returns is evidence supporting the price discovery hypothesis that new market information disseminates in the futures markets before the stock market, with index arbitrageurs stepping in quickly to bring the cost of carry back to alignment' (Stoll & Whaley, 1990: 466).

In deriving the theoretical futures prices, the cost of carry model does not take into account the transaction cost of the element in the arbitrage strategy. In reality, the cost of entering into and closing cash positions, together with the round-trip transaction cost for the futures contract, does affect the futures prices. Transaction costs have the effect of widening the boundaries for futures prices. MacKinlay and Ramaswamy (1988) believe that the impact of the transaction cost is to allow futures prices to fluctuate within the width of a band around the cost of carry price. According to them, the width of the band derives from the round-trip commissions in the futures and stock market and from the cost of putting on the trade initially. Brennan and Schwartz (1990) had the same to say about transaction costs.

Koutmos and Turker (1996) believe that the futures market is more efficient than the spot market for two reasons:

- Most shares in the index do not trade at different prices at all times and, as a result, the index responds to the new information with a lag.
- Lower transaction costs in the futures market make it advantageous for investors with strong beliefs about

the direction of the market to trade index futures instead of the index itself.

As a result, movements in the futures prices will lead movements in the spot prices.

Sim and Zerbreugg (1999) studied the intertemporal effects of foreign share and futures market on the domestic spot-futures relationship. They concluded that one primary reason for the difference between the spot market and the futures market is that the futures market has fewer trading constraints than the spot market. Consequently the futures market is more information efficient than the spot market.

Stoll and Whaley (1990) examined intraday changes from the S&P 500 and MM share indices and futures contracts for serial correlation. They noticed that the futures market leads the share market by an average of five minutes. According to them, the fair price is different from the cost of carry value because:

- market-wide information is systematic in nature and would feed through the futures market before entering the spot market, and
- the infrequent trading of shares within the index market is not perfectly continuous.

Rolph (1999) examined how well the Federal funds futures rate predicts changes in the funds target rates relative to the spot rate predicted. He hypothesised that the futures market has meaningful information about changes in monetary policies, and found that futures rates have greater information relative to the spot rate in predicting future changes in monetary policy.

Chen *et al.* (1995) developed a model, hereafter called the CCH model, to analyse the relationship between the spot and futures markets. Their model explains deviations of the observed future prices from the value predicted by the cost of carry model by analysing the basis. The basis should be negatively related to the return volatility of the underlying asset. They tested their model on data of S&P 500 contracts in the USA and found that there were some systematic deviations from the cost of carry model, which arose when investors experienced substantial differences between the two markets. In their model, investors differ with regard to the customisation value (net advantage of stock compared to futures) they attach to their assets. Since having a customised value is specific for assets and not for futures position, investors faced with an increase in the market risk tend to adjust the risk of their assets by adjusting their futures position rather than by selling the assets. This implies that futures prices will drop relative to the spot prices when market volatility increases.

This is consistent with the findings of Berglund and Kabir (1995) who found that when the volatility of the underlying asset is high, the futures prices tend to be lower relative to the expected spot price. They used the CCH model to analyse systematic deviations of the observed futures prices from the value predicted by the cost of carry model. They

found support for their model on data for stock index futures on the European Options Exchange in Amsterdam.

The data

The following data series are used in the study:

- daily closing prices for futures contracts on the three main share indices of the South African Futures Exchange (ALSI, INDI and FINI);
- daily spot prices of the indices;
- daily dividends paid on the shares of each index;
- volatility of the underlying assets of each index;
- the basis;
- daily risk-free rate of return; and
- open interest

The data were obtained from different sources. The closing prices of futures, contracts, daily spot prices of the underlying indices, volatility of the index and open interest of the contracts were all obtained from the Johannesburg Securities Exchange (JSE). The bankers' acceptance (risk-free rate) data and the daily dividends were obtained from the I-NET database.

In the case of all three indices, prices of futures contracts with the nearest expiry date, i.e. the most actively traded contracts, were used. Data used for the futures markets are the daily settlement prices for the indices traded on the South African Futures Exchange (SAFEX), and the delivery months are March, June, September and December, on the third Thursday of the month.

In measuring the volatility of the underlying index, implied volatility of prices for traded options on the indices are used. The implied volatility is the trading volume weighted averages of the standard deviations computed with the Black and Scholes formula for the near-the-money call and put contract on the indices. The volatility has been computed by SAFEX. The period of analysis is from 4 January 1998 to 31 December 2001, giving a total of 64 non-overlapping contracts and 972 observations for each index.

Open interest tends to increase as time to maturity decreases. This trend is repeated every three months, since there are four maturity dates in a year. In order to accommodate this seasonal phenomenon a dummy variable *D* is included in the regression equation. *D* takes the value of unity on each maturity date and is zero on any other day. In doing so, the constant is allowed to vary on all maturity dates. The dummy variable for March delivery in each of the four years was not defined, because normally first-quarter constants are used as a benchmark against which changes in other delivery dates are measured.

The following dummy variables were defined:

$D_2 = 1$ at June delivery, $D_2 = 0$ on other days

$D_3 = 1$ at September delivery, $D_3 = 0$ on other days

$D_4 = 1$ at December delivery, $D_4 = 0$ on other days.

Empirical results

ALSI results

Autocorrelation analysis shows that no significant autocorrelation is present after the second lag, therefore the first two lags of the basis were included as independent variables in the regression equation.

A number of researchers have shown that, more often than not, prices in the futures markets lead those in the cash market. To accommodate this effect, daily returns, both contemporaneous and with a one-day lead, are included in the regression equation. This controls for the effect of possible non-synchronous changes in the futures markets and in the index itself.

Table 1: Regression summary of implied volatility on basis – ALSI

	Coefficient	p-value
Intercept	-2,20	0,028
Vol	-0,00	0,803
ALSI _t	237,27	0,000
ALSI _(t+1)	48,58	0,408
Basis _(t-1)	0,40	0,000
Basis _(t-2)	0,27	0,00

Adjusted R² = 0,369

The results are shown in Table 1. The volatility of the underlying index (Vol) has a negative slope coefficient and a p-value of 0,803. The result indicates a negative relationship between the basis and volatility. However, the relationship is weak and insignificant.

The lagged basis variables, Basis_(t-1) and Basis_(t-2) have positive slope coefficients with associated p-values of 0,000. This is a reflection of the tendency of the basis to persist for a few days.

The ALSI returns, both contemporaneous and with a lead, have a positive relationship with the basis. The instantaneous effect of share returns on the basis is clearly significant.

Table 2 shows 5 different regression specifications using futures contracts closest to maturity. Using these futures contracts helps to reduce problems associated with thin trading. Consequently, more significant results than those in Table 1 are expected, as more actively traded futures contracts are used.

In the first regression volatility is regressed against the basis. Volatility has a slope coefficient of -0,154 and a t-statistic of -0,97. Although not significant, this is in agreement with the prediction of the CCH model that an increase in volatility will lead to a decrease in the basis.

Table 2: Daily regressions of implied volatility on basis – ALSI

Equation	1	2	3	4	5
Basis _(t-1)	0,007 (0,07)	-0,030 (-0,34)	-0,014 (-0,21)	-0,023 (-0,24)	-0,018 (-0,18)
Basis _(t-2)	-0,024 (-0,03)	-0,01 (-0,08)	-0,001 (-0,18)	0,002 (0,02)	-0,010 (-0,11)
ALSI	-3,119 (-0,09)	18,740 (-0,06)	-1,914 (-0,06)	1,703 (0,46)	-6,090 (-0,18)
ALSI _(t+1)	-721,116 (-3,10)	-816,120 (0,53)	-819,197 (-3,61)	-840,145 (-3,78)	-792,108 (-3,58)
σ^2	-0,154 (-0,97)	-0,160 (-0,15)	-29,196 (-0,57)		
$\sigma^2 \tilde{t}$			-0,101 (-29,20)	-9,463 (-3,01)	-26,422 (-3,09)
\tilde{t}	-	-1236,240 (-3,39)	-	-905,193 (-1,47)	-
Intercept	-20,928 (-2,46)	-12,850 (-1,52)	-23,611 (-2,80)	-18,635 (-2,90)	-20,573 (-3,26)
Observations	110	110	110	110	110
R ²	0,128	0,216	0,196	0,211	0,194
Adjusted R ²	0,086	0,170	0,149	0,165	0,155

The dependent variable is the computed basis. ALSI is the contemporaneous return of the ALSI cash index. σ^2 is the implied volatility of the underlying index. \tilde{t} is time to maturity. t- statistics are in parenthesis. The sample period is from January 1998 to December 2000.

In regression two, the slope coefficient of the volatility is also negative, with a t-value of -0,15. This again shows that there is evidence of a negative relationship between the basis and the volatility. The slope coefficient for time to maturity is also negative, with a t-statistic of -3,39. This shows that there is overwhelming evidence to infer that the basis is negatively related to time to maturity. This is so because, on average, the basis is negative. As time to maturity decreases, the basis increases from being negative to a value of zero at maturity.

The CCH model suggests that the basis response to the change in volatility should be scaled by time to maturity. To account for that, equations three and four have a variable that is a product of volatility and time to maturity ($\sigma^2\tilde{i}$). In both equations $\sigma^2\tilde{i}$ has a strong, negative and significant relationship with the basis. In regression three, the slope coefficient of $\sigma^2\tilde{i}$ is negative with a t-statistic of -29,20. This indicates a strong negative relationship between the basis and volatility as implied by the CCH model. The results in regression four are the same, with volatility having a slope coefficient of -9,463 and a t-value of -3,01.

In regression five, the results show a slope coefficient of -26,422 and a t-statistic of -3,09. There is overwhelming evidence to infer that the basis is negatively related to the volatility of the underlying asset.

It is worth noting that the relationship between the basis and volatility of the underlying asset becomes stronger when volatility is measured as a product of volatility and time to maturity than when it is not.

Table 4: Daily regressions of implied volatility on basis – FINI

	1	2	3	4	5
Basis _(t-1)	0,005 (3,76)	0,005 (3,67)	0,005 (3,69)	0,636 (6,02)	0,005 (3,68)
Basis _(t-2)	0,000 (0,32)	0,000 (0,33)	0,000 (0,33)	-0,075 (-0,72)	0,001 (0,42)
FINI	0,409 (0,417)	0,354 (0,35)	0,395 (0,39)	0,078 (0,91)	0,279 (0,27)
FINI _(t+1)	0,215 (0,37)	0,217 (0,37)	0,216 (0,37)	-0,047 (-0,56)	0,307 (0,53)
$\sigma^2\tilde{i}$	-	-	-0,014 (-0,02)	0,16 (0,59)	0,065 (0,35)
σ^2	0,009 (1,32)	0,009 (1,30)	0,009 (1,27)		-
\tilde{i}	-	-2,903 (-0,40)	-	-0,126 (-0,46)	-
Constant	0,731 (3,00)	0,776 (2,87)	0,732 (2,95)	3,590 (0,40)	1,017 (9,42)
Observations	99	99	99	99	99
R ²	-	0,1814	0,1799	0,3607	0,1651
Adjusted R ²	-	0,1087	0,1070	0,3190	0,1009

The dependent variable is the basis. FINI is the contemporaneous return of the FINI cash index. \tilde{i} is the time to maturity. σ^2 is the volatility of the underlying asset. t-statistics are in parenthesis. The sample period is from January 1998 to December 2001.

FINI results

The CCH model's predictions are also tested on the FINI futures contract. Two lagged variables for the basis were included to take care of serial correlation problems. The two FINI return variables were included to reduce or control for non-synchronous changes in both the futures price and the index price.

The results are shown in Table 3.

Table 3: Regression summary of implied volatility on basis – FINI

	Coefficient	p-value
Intercept	1,37	0,000
Vol	-0,00	0,185
FINI _t	-2,18	0,001
FINI _(t+1)	0,84	0,289
Basis _(t-1)	0,00	0,000
Basis _(t-2)	0,00	0,093

Dependent variable = Log Basis

Adjusted R² = 0,065

The results confirm the CCH model's prediction that the basis is negatively related to the volatility of the underlying asset. The slope coefficient of the volatility is -0,003, which indicates that there is a negative relationship between the basis and the volatility of the underlying asset, although it is not significant. Again the effect of the basis persists in time.

The experiment was repeated using futures contracts that were near maturity. The results are reported in Table 4.

For all five equations, the first lagged variable is positive and significant. This implies that the effect of the basis has a tendency to persist for at least one day. Both FINI returns tend to be positively related to the basis. It shows that FINI futures prices have a tendency to lead the cash market. The coefficient for time to maturity is negative, indicating a negative relationship between the basis and time to maturity. As time to maturity decreases, the basis will increase to zero on delivery date.

Here volatility is positively related to the basis. This result in general is repeated for time-scaled volatility. The reason for this peculiar result may be due to the fact that the FINI is thinly traded. In fact, for most of 1998 the FINI did not trade at all. As a result, volatility of the underlying asset becomes very stable. For example, the volatility for FINI index stayed at 48% for the 3-month period 18 December 1998 to 18 March 1999.

INDI results

The results of INDI regression are shown in Table 5.

Table 5: Regression summary of implied volatility on basis – INDI

	Coefficient	p-value
Intercept	10,57	0,022
Vol	-0,31	0,032
INDI _t	-81,91	0,229
INDI _{t+1}	-422,83	0,000
Basis _{t-1}	0,37	0,000
Basis _{t-2}	0,24	0,000
Basis _{t-3}	0,12	0,000

Adjusted R² = 0,434

The results are consistent with the CCH model prediction. The volatility slope coefficient is negative 0,311, with a p-value of 0,032. There is strong evidence to infer that the basis is negatively related to the volatility of the underlying index.

The lagged basis variables all have positive and significant slope coefficients indicating the effect of the basis trends to persist for a few days.

Again five different equations are run for contracts closest to maturity and all show the same result: volatility is positively, but insignificantly related to the basis and time to maturity. The results appear in Table 6.

Table 6: Daily regressions of implied volatility on basis - INDI

Equation	1	2	3	4	5
Constant	-37,543 (0,32)	-44,529 (0,30)	-39,175 (0,30)	-22,249 (0,33)	-23,35 -1,17
INDI	-0,088 (0,47)	-0,079 (0,48)	-0,084 (0,45)	-0,076 (0,51)	0,073 (-0,67)
INDI _(t+1)	-0,088 (0,47)	-0,013 (0,90)	-0,016 (0,88)	-0,021 (0,85)	-0,021 (-0,19)
Basis _(t-1)	0,005 (0,97)	0,000 (0,99)	0,000 (0,99)	0,002 (0,98)	0,002 (0,01)
Basis _(t-2)	-0,06 (0,58)	-0,063 (0,56)	-0,063 (0,57)	-0,062 (0,57)	-0,063 (-0,58)
Basis _(t-3)	-0,00 (0,97)	-0,006 (0,95)	-0,004 (0,97)	-0,003 (0,98)	-0,004 (-0,04)
σ ²	0,070 (0,51)	0,074 (0,51)	0,059 (0,62)		
σ ² \tilde{i}			0,044 (0,71)	0,075 (0,63)	0,063 (0,57)
\tilde{i}		0,0390 (0,72)		-0,0161 (0,91)	
R ²	0,1255	0,0140	0,0142	0,0115	0,0113
Adjusted R ²	–	–	–	–	–
No. of observations	92	92	92	92	92

The dependent variable is the computed basis. INDI is the contemporaneous return of the INDI cash index. \tilde{i} is the time to maturity. σ² is the implied volatility. t-statistics are in parenthesis. The sample period is from January 1998 to December 2001.

Equation two includes the time-to-maturity variable. The results show that the basis has a positive relationship with time to maturity, i.e. as time to maturity decreases to zero, so does the basis. It should be noted that in the INDI the futures prices on average are above the fair price. As a

result, the basis is positive on average (the mean is 4.0681). The positive relationship between the basis and time to maturity shows that as the basis (which is positive) decreases to zero at maturity, the time to maturity will also decrease to zero at maturity. This also explains the sign of

the volatility variable which does not conform to theoretical expectations.

In equation four, a product of time to maturity and volatility was included in the regression equation since the basis response to changes in volatility is scaled by time to maturity. The results still show a negative relationship between the basis and volatility.

Basis and interest rate

The final step in the analysis of the basis is to find the relationship between the basis and the interest rate. Helmer and Longstaff (1991) predict that the basis has a negative concave relationship with the interest rate. The impact of interest rate on volatility will be measured by time to maturity. The longer the remaining time to maturity, the greater the impact. As a result, the product of time and maturity and risk-free interest rate are included in the regression equation.

Because it is predicted that there is a concave relationship between the basis and open interest, a term containing the squared interest rate was also included to test for a non-

linearity in the relationship. The coefficient of $r^2 \tilde{I}$ will describe the curvature. If the coefficient is zero, it means that there is no curved relationship between variables. If the coefficient is negative, then the relationship is concave, if positive, the relationship is convex. The greater the value of the coefficient, the greater the rate of the curvature.

The following regression equation was used:

$$Y = \alpha + B_1X_1 + B_2X_2 + B_3X_3 + B_4X_4 + \epsilon$$

where:

$$\begin{aligned} Y &= \text{basis} \\ X_1 &= r \cdot \tilde{I} ; \\ X_2 &= r^2 \tilde{I} ; \\ X_3 &= \sigma \tilde{I} ; \text{ and} \\ X_4 &= \sigma^2 \tilde{I} \end{aligned}$$

The results are reported in Table 7.

Table 7: Daily regressions of basis on interest rate

	α	$\sigma \tilde{I}$	$r^2 \tilde{I}$	$r \cdot \tilde{I}$	$\sigma^2 \tilde{I}$	Observations
ALSI	-20,960 (-9,72)	-5,371 (-2,50)	-22943,000 (-9,42)	5783,700 (2,50)	0,002 (2,50)	974 -
INDI	-21,59 (-7,850)	-51,80 (-3,600)	-4726,900 (-1,340)	3306,400 (0,240)	-0,726 (-3,330)	976 -
FINI	7,090 (2,130)	-32,300 (-2,700)	-43204,300 (-3,860)	9802,220 (3,180)	0,544 (3,180)	978 -

The dependent variable is the computed basis. σ is the implied volatility of the underlying index. \tilde{I} is the time to maturity of futures contracts and r is the daily risk-free interest rate. t-statistics are in parenthesis. The sample period is from January 1998 to December 2001.

The results are in agreement with the findings of Helmer and Longstaff (1991). The basis has a negative, concave relationship with the interest rate.

For ALSI, the coefficient for $r^2 \tilde{I}$ is -22943,74 with a t-statistic of -9,42. This shows that there is a significant concave relationship between the interest rate and the basis.

The coefficient for $\sigma \tilde{I}$ is also negative, confirming the prediction of Chen *et al.* (1995) that the basis is negatively related to the volatility of the underlying index.

FINI shows similar results, and the basis has a negative concave relationship with the interest rate. The $r^2 \tilde{I}$ coefficient is -43204,3 with a t-statistic of -3,86. This shows that there is overwhelming evidence to infer that the basis has a concave relationship with the interest rate.

The results are similar for the INDI variable.

Volatility and open interest

A further prediction of the Chen *et al.* (1995) model is that volatility is positively related to the open interest of the

futures contract. To investigate whether this prediction is supported by the data, the regressions reported in Tables 8 to 10 were estimated. Time to maturity as variable was included in order to allow for the cumulative increase in the open interest as maturity approaches. As in the case of the CCH model, volatility was measured as a product of volatility and time to maturity.

Table 8: Regression summary of volatility on open interest – ALSI

	Coefficient	p-value
Intercept	7508,3	0,031
$\sigma \tilde{I}$	25,70	0,888
\tilde{I}	-953,10	0,974
OI_{t-1}	0,20	0,000
OI_{t-2}	0,20	0,000
OI_{t-3}	0,20	0,000
OI_{t-4}	0,10	0,001
OI_{t-5}	0,10	0,025
OI_{t-6}	0,10	0,031
OI_{t-7}	0,10	0,037
D	-53707,7	0,000

Adj R² = 0,445

The results indicate that open interest tends to increase with increased volatility. The scaled volatility slope coefficient is 25,70, although not significant. The lagged open interest variables are all positive and significant, demonstrating the carry-over effect of open interest.

Table 9: Regression summary of volatility on open interest – FINI

	Coefficient	p-value
Intercept	-34,78	0,139
$\sigma \tilde{I}$	25,71	0,016
\tilde{I}	-226,39	0,556
OI_{t-1}	0,72	0,000
OI_{t-2}	0,21	0,000
OI_{t-3}	0,04	0,156
D	-704,89	0,000

Adj R² = 0,952

Table 10: Regression summary of volatility on open interest – INDI

	Coefficient	p-value
Intercept	-108,71	0,638
$\sigma \tilde{I}$	93,58	0,266
\tilde{I}	6459,50	0,028
OI_{t-1}	0,62	0,000
OI_{t-2}	0,25	0,000
OI_{t-3}	0,10	0,001
D	-7045,94	0,000

Adj R² = 0,917

These results are confirmed in the case of the FINI index (Table 9) and the INDI index (Table 10).

Summary and conclusion

The article investigates the relationship between the basis and volatility of the underlying asset and between the basis and open interest. The relationship is investigated using the three South African indices, the ALSI, FINI and INDI, which are all trading on SAFEX. The sample included data from January 1998 to December 2001.

The results correspond to those obtained by Chen *et al.* (1995) in that the basis is negatively related to the volatility of the underlying asset. This is consistent with the view of Chen *et al.* (1995) that when the volatility of the underlying asset increases, the market becomes riskier and as a result the expected rate of return increases. This will lead to an increase in the share price relative to the futures price, and consequently the basis will decrease.

On the other hand, if market volatility decreases, the shares will be less risky and the expected rate of return will decrease. This will make shares less attractive to investors and the spot or index price will therefore decline relative to the futures prices. As the index prices decline, the basis will increase.

The other main prediction of the CCH model which is supported by the study is that the open interest in futures contracts is positively related to the volatility of the underlying asset. This prediction was also tested on three South African indices trading on SAFEX, namely the ALSI, FINI and INDI. The results are also consistent with the predictions of the CCH model. As the volatility of the underlying index increases, so does the open interest of the futures contracts. As volatility increases, investors would like to hedge their stock positions by buying futures. As short positions increase, so does the open interest of the futures contract.

On the other hand, in cases where volatility decreases, long positions of futures contracts will increase relative to the short positions. As a result open interest will decrease.

Like Helmer and Longstaff (1991), the study also tested the relationship between the interest rate level and the basis. The hypothesis was that the interest rate level has a negative concave relationship with the basis. This prediction was tested on the three indices ALSI, FINI and INDI. All the results show that there is a highly significant negative relationship between the interest rate level and the basis. In cases where the interest rate is high, investors will buy futures and invest in a risk-free instrument. As a result, the futures price will go down relative to the spot price and the basis will decrease. On the other hand, a lower interest rate level will produce a higher futures price relative to its fair value. This will result in an increase in the basis.

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