COMPUTERIZED JOBSHOP LAYOUT PLANNING

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ABSTRACT

The multitude of feasible arrangements of work centres in a jobshop can be viewed as a finite statistical population. The model presented here uses computer simulation to generate and evaluate samples from this population, with the objective of obtaining some observations from the vicinity of the over-all optimum. The practical importance of layout planning follows from two conditions, viz. the omnipresence of jobshops in industrialized countries such as S.A., and the high portion of manufacturing costs which may in these instances be ascribed to materials handling, i.e. directly to the quality of the plant layout. Accordingly, this model was designed to satisfy a very real practical need.

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1. Introduction

Layout planning can be described in general terms as the task of assigning relative locations to a set of facilities, such that a given level of transactions between these may be carried out with a maximum over-all efficiency. With particular reference to **jobshop layout planning**,. this problem area has received considerable attention during the past decade [5]. The continuing interest in layout planning could perhaps be attributed to two major causes, i.e. a growing awareness of the role of materials handling as an important factor contributing to manufacturing costs [1], and the absence of an optimizing planning technique. This interest has stimulated the development of heuristic models such as ALDEP [12], CORELAP [7,8], CRAFT [2,3], MAT [4], and RMA Comp I [10]. The practical and theoretical ramifications of jobshop layout and facilities location are discussed in detail elsewhere [2,5,10,13]. It would therefore be appropriate to restrict this introduction to a brief comparison of the aforementioned heuristic models and an examination of two contentious aspects of layout planning.

1.1 Computerized Layout Planning Models

All of the aforementioned models are computer-based, and are aimed at placing facilities in a confined space, where the ground plan is either given or to be defined subsequently. As such they are, at least in theory, applicable to a wide variety of layout problems, including plants, office buildings, department stores, industrial sites and computer centres.

CRAFT uses an initial layout plan, which forms part of the program input, as a starting point for an iterative procedure: The "current plan" is scanned exhaustively for feasible exchanges of two and/or three facilities at a time. The most promising exchange is then implemented, thus forming a new "current plan". An exchange is feasible if the facilities concerned are either of equal size or occupy adjacent areas. When no further profitable exchanges of two or three facilities can be detected, the "current plan" becomes the "final plan". Returns of the model with different initial plans are advocated since the deterministic search criterion establishes a

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[1] In (generally labor-intensive) jobshops, it is not uncommon for the cost of materials handling, a derivative of the layout, to reach 30 or 40 percent of the cost of direct labor, [14]. Moreover, approximately three-quarters of a large sample of interviewed US manufacturing companies considered themselves engaged in some form of intermittent production (source: F.G. Moore and R. Jablonski, **Production Control**, 3rd ed., McGraw-Hill, 1969, p. 19).

unique, deterministic relationship between each pair of initial and final plans.

CORELAP constructs layout plans in a crystal-growth fashion, the facility with the most interactions being placed in a centre location. Subsequently, facilities are selected and placed on a basis of their transactions with the growing layout plan. By contrast, RMA Comp I develops a schematic pattern of relative locations first; then assigns the required spaces to facilities without regard to layout continuity ("fragmentation"). Based on a near-optimizing mathematical algorithm, MAT operates in a very similar manner — a continuous plan is constructed while assuming that identical spaces will be allocated to all facilities.

All four models only generate a single final plan. ALDEP provides several alternatives: The first facility to be placed is selected randomly, while subsequent facilities are selected and fitted as in CORELAP, with the proviso that random selection takes over again where no significant priorities exist. It will be seen later on that although it was developed along different lines originally, the current form of the simulation model to be presented here shares several characteristics with ALDEP.

By way of a preliminary summary, (a) only one of the five models (CRAFT) modifies an initial plan while all others construct new plans; (b) two models (RMA and MAT) constructed schematic layout plans while the other three generate detailed two-dimensional plans; (c) of the latter three models, only CRAFT and ALDEP are designed to operate withing the specified ground plan of a building or site; and (d) only ALDEP generates several alternative plans in a single run while of the other four only CRAFT can provide a choice by means of reruns each commencing with different initial conditions.

1.2 Objective Criteria

In a jobshop, the aforementioned "transactions" usually take the form of materials flows between work centres. This notion is easily extended, however, to include personnel trips to and between service facilities, or to express personnel trips only. Taken in this wider sense, then, the concept of "flows" between "work centres" or departments can be used to evaluate both industrial and non-manufacturing layouts [13] an approach which was adopted in CRAFT and MAT. Such flows are aggregated for a representative period of time; total materials handling cost is then computed for that period as a linear function of aggregate flows, distances between work centres, and unit cost factors which reflect the handling equipment used 1). Flow volumes and cost factors may respectively represent actual or hypothetical shop loads and handling equipment while the distances between work centres are a derivative of the layout which is accordingly constructed or altered so as to near-minimize period handling costs.

An alternative criterion is due to MUTHER [9]. Arguing that it may be difficult or impossible to obtain reliable flow and/or cost data, and that in some instances flow costs may not be the predominant criterion, MUTHER has developed a list of closeness ratings ranging from "absolutely necessary" to "undesirable". Each pair of departments is assigned such a priority of closeness, and layouts are evaluated in terms of total closeness ratings realized. This technique — which owes its name to the REL (ationship) chart of assigned closeness ratings — is used in ALDEP, CORELAP and RMA.

For two reasons, the authors prefer the handling cost criterion where a choice exists. Firstly, this criterion focusses on the most important single cost factor associated with industrial layout. Secondly, it facilitates an analysis of layout sensitivity with respect to changes in flow patterns and modifications of handling systems and/or equipment 2). The latter is particularly important in view of the static quality of a layout versus the dynamic nature of the operations (flows) which it must accommodate efficiently. Conditions which would give rise to "absolutely necessary" or "undesirable" ratings in a REL chart should be formulated as con-

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1) Non-linear functions could be used in CRAFT and MAT as well as in the simulation model; see 3.2.

2) Results which emphasize the importance of such analysis are reported in [13].

straints, to be imposed on the cost oriented criterion function. However, in recognition of the fact that neither criterion is universally applicable, it is felt that layout planning models should provide optional criteria, including pure cost functions and closeness ratings as well as intermediate forms such as a cost function which is constrained by the mandatory REL chart ratings.

1.3 ground Plans

LEE and MOORE, authors of CORELAP [7], argue that a preconceived building ground plan imposes severe restrictions on the layout, and that this makes no sense whatsoever if a brand new plant is to be designed. Accordingly, CORELAP plans expand freely in all directions, guided by the objective criterion and restricted only by an over-all length-to-width ratio. There can be no doubt that unsuitable ground plans must have an adverse effect on layout efficiency. The question, however, remains whether a preconceived plan must of necessity be unsuitable, or more precisely, whether it might not be possible — and in fact desirable — to predetermine a suitable ground plan. Experiments with the simulation model, in which layouts were generated, under otherwise equal conditions, for rectangular 1:1 and 3:2 ground plans, indicated handling cost advantages of between five and ten percent for the square pattern. Supplementing this result, compact, near-square ground plans also offer **a priori** advantages in terms of plant space and plant site utilization as well as plant construction and maintenance costs. Concluding from this and further analysis [14,pp. 42 ff], the simulation model was designed to operate within the framework of given (existing or hypothetical) building ground plan mappings. Furthermore, near-square patterns are advocated as a general guideline for the design of new plants, notwithstanding the possibility of overriding considerations other than handling cost, space utilization, and construction and maintenance costs.

2. The Rationale of Layout Simulation

Any arrangement of **n** work centres1) in a specified ground plan G can be described in terms of a congruent set L which is an ordered union of **n** non-empty and disjunct subsets x_i,

$$L = \bigcup_{i=1}^{n} x_i, \qquad (2.1)$$

$$\text{subject to } L \equiv G \text{ (congruence)}$$

$$x_i \neq \emptyset$$

$$x_i \cap x_j = \emptyset,\ i \neq j\ i,j = 1,2,\ldots,n$$

if all x_i are identical moduli,

$$x_i = x_j\ ;\ x_i \equiv x_j\ ;\ i \neq j\ ;\ i\,j = 1,2,\ldots,n,$$

the number of materially different arrangements L is given by

$$N = \begin{cases} n!, \text{ for } s = O \\ n!/s, \text{ for } s > O \end{cases}$$

where **s** denotes the degree of symmetry of G, see [2,p.297]. Although very large already, N is further increased if the condition of modular congruence is relaxed and $x_i \not\equiv x_j$ is premitted, which is inevitable where work centres with widely varying floor space requirements, $x_i \neq x_j$, must be accommodated.

In the absence of tractable optimization techniques, it appears conceptually attractive practically advantageous to consider the multide of possible arrangements L as a statistical population from which samples can be drawn with a view to obtaining some observations from the vicinity of the population optimum. This implies repetitive construction of independent plans rather than repeated modifications of an initial layout that could prejudice the results. This aspect is of particular importance in multi-floor planning with non-identical x_i Furthermore, because it does not use a systematic local search criterion, the evaluation of a series of independent observations commands a better **a priori** chance of detecting truly near-optimal solutions. Such guidance as is required to increase the efficiency of this — non-path oriented and potentially global — search will have to be provided by biased sampling techniques.

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1) The term "work centre" is used in a wider sense to comprise all organizational units serving as origins and/or destinations for internal traffic. In a jobshop situation, this includes service and recreational facilities, elevators, etc., but not aisles or lanes which are assumed to be part of each work centre and taken into account in its floor space requirements.

The flow chart in Fig. 1 illustrates the basic structure of the simulation model which was designed to implement this approach.

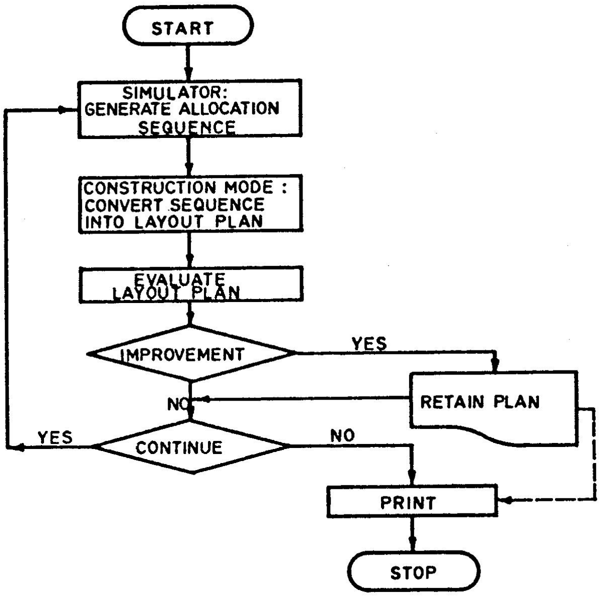


FIGURE 1: SCHEMATIC FLOW CHART OF THE SIMULATION MODEL.

Each loop through the diagram produces one observation: The **simulator** generates a random or biased-random sequence of work centre identification numbers **i**; a **construction mode** converts this sequence into a two-dimensional layout plan L; an evaluation routine measures the efficiency of this plan; and subsequent checks control intermediate storage and program termination.

The primarily theoretical problems which the simulator presents in respect of sampling efficiency and sample sizes can be solved by means of statistical analysis and experimentation. The technical difficulties encountered in the development of a suitable construction mode are considerable: While there is no need for layout plans to be immediately applicable as blueprints for construction work, they must be sufficiently realistic to require only minor adjustments. If major corrections are necessary, the efficiency gains suggested by such plans may be fictitious since "gains" which were derived under unrealistic assumptions as to what constitutes a practicable solution, are meaningless. Still, such demands imply a rather formidable conceptual and programming task, and to what extent this has been solved here remains open for discussion.

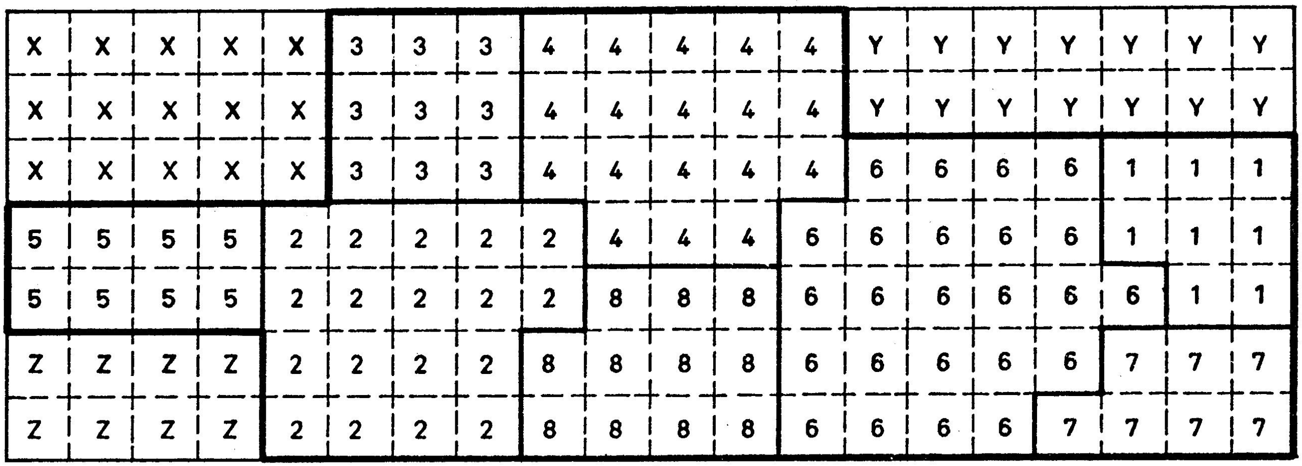
3. Layout Construction and Evaluation

For purposes of layout plan construction and storage thereof in a digital computer it is useful to define

(a) a two-dimensional modulus such that all work centres and the ground plan G can be expressed as integer multiples of this basic unit area; and

(b) an allocation matrix A whose elements shall represent one modulus each, dimensioned such that it can accommodate the ground plan in modular form.

Any arrangement L may then be constructed



X	X	X	X	X	3	3	3	4	4	4	4	4	Y	Y	Y	Y	Y	Y	Y
X	X	X	X	X	3	3	3	4	4	4	4	4	Y	Y	Y	Y	Y	Y	Y
X	X	X	X	X	3	3	3	4	4	4	4	4	6	6	6	6	1	1	1
5	5	5	5	2	2	2	2	2	4	4	4	6	6	6	6	6	1	1	1
5	5	5	5	2	2	2	2	2	8	8	8	6	6	6	6	6	6	1	1
Z	Z	Z	Z	2	2	2	2	8	8	8	8	6	6	6	6	6	7	7	7
Z	Z	Z	Z	2	2	2	2	8	8	8	8	6	6	6	6	7	7	7	7

FIGURE 2: BLOCK LAYOUT PLAN.

and stored in matrix form as shown in Fig. 2, where work centres **x, y,** and z are **dummies** used solely for purposes of delineating G in A. Having no flow connections with any of the real work centres **i** = 1,2,...,8, these dummies constrain the layout construction without entering the simulation or evaluation processes.

3.1 High-Speed Repetitive Layout Construction

A layout plan shall be called **feasible** if it meets the requirements specified in (2.1), assigns a continuous space of x_i moduli to each work centre **i**, and shows an acceptable configuration of moduli in each instance[1] Planning economy dictates that a construction mode should generate a high percentage of feasible layouts with a minimum of computer time. These conditions point to a highly systematic construction mode, a class of which are shown in Fig. 3. While all three types of movements,
i.e. parallel, single-, and double-oscillatory, are well-suited for computer coding and high-speed execution, only the two modes of double-oscillatory matrix traversion were found to perform satisfactorily in terms of the aforementioned requirements, [14]. The functioning of (x,y) - oscillation is illustrated in Fig. ê, where only work centres 1,5,12, and 20 are shown while, of course, the entire ground plan is covered by a complete layout plan.

		DIRECTION OF X.	
		NOT CHANGING	CHANGING
DIRECTION OF Y.	NOT CHANGING	PARALLEL	X- OSCILLATORY (SINGLE)
	CHANGING	Y- OSCILLATORY (SINGLE)	(X, Y)- AND (Y,X)- OSCILLATORY. (DOUBLE)

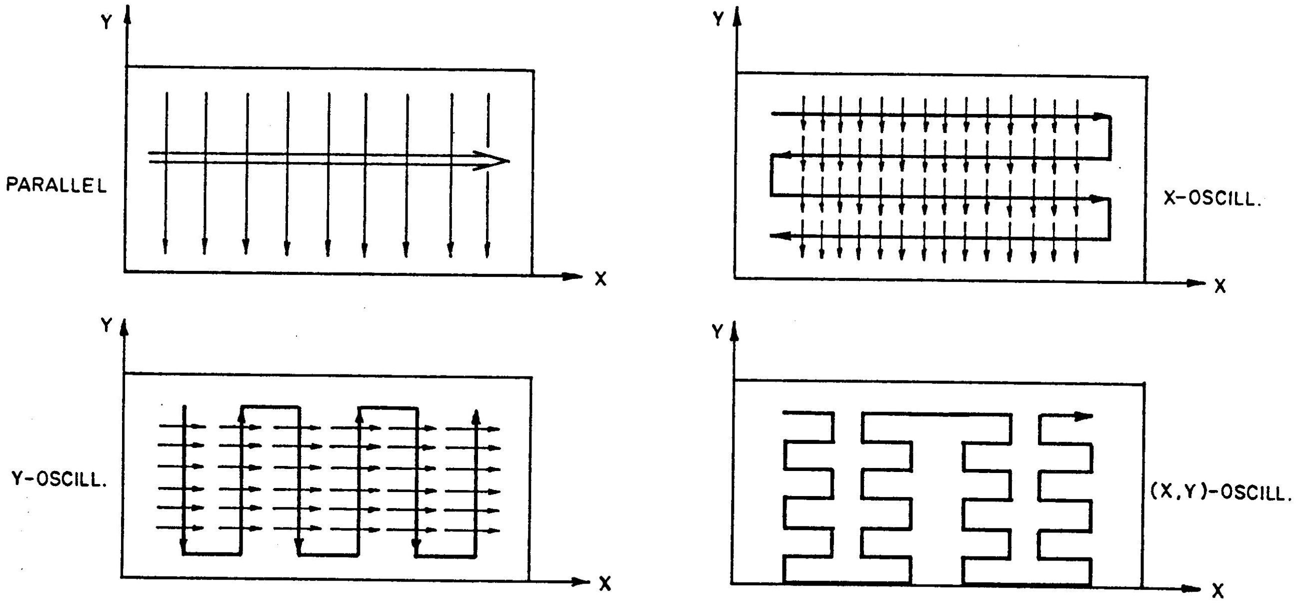


FIGURE 3: A CLASS OF SYSTEMATIC CONSTRUCTION METHODS.

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[1] "acceptability" of work centre contours is measured in terms of departmental space utilization, facility of internal operations, and inner-departmental materials handling economy. Basically, compact or near-square contours are acceptable while, in this general context, I-, L-, T-, or U-shapes are not.

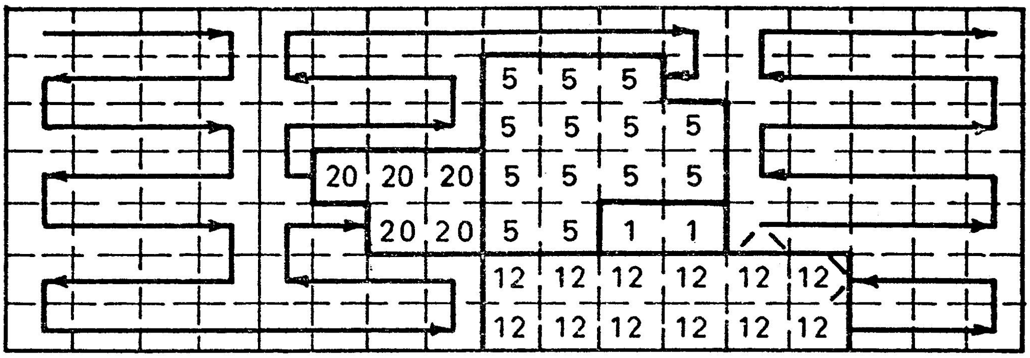

FIGURE 4: (X,Y)-OSCILLATORY CONSTRUCTION METHOD.

Oscillatory construction modes require a subdivision of the allocation matrix into sections consisting of several rows or columns each — see Figs. 3 and 4. The width of each section determines the maximum width of depth of all departments placed in it, and in that sense exerts a standardizing influence on the contours assigned to work centres. The width of a **standard section** is given by the rounded square-root of the mean $x = \frac{1}{n} \Sigma n = 1\ x_i$, following the notion that near-square configurations are **a priori** preferable. Experiments with variable section widths confirmed the expectation that department contours would become unacceptable in the aforementioned sense, thereby discounting the small gains in efficiency indicated by the objective function.

Two problems arise from the use of sections, standard or otherwise. Firstly, an "overflow" will occur when the last work centre allocated to one section must be carried over into the adjacent section because of insufficient space in the former, a case which can be considered the rule rather than an exception. Work centre 12 in Fig. 4 shows how this problem is solved: The incomplete department is extended, over its full depth, into the adjacent section before the oscillatory routine takes over again.

Secondly, dummies and fixed departments, neither of which may be removed from their prescribed locations in A, form obstacles in the path of the allocation process, impairing its continuity. If disrupted, the process will split departments into disjoint fragments, thereby rendering entire layout plans infeasible. Disruptions can occur as a result of several sets of conditions, three of which are shown in Fig. 5 together with appropriate remedial actions. In general, continuity can be ensured through (a) control of the initial direction — positive or negative — of x and y; (b) intermediate change of directions after the completion of any work centre; and (c) the use of predesigned sections which form part of the program input rather than being developed internally as indicated above. Controls (a) and

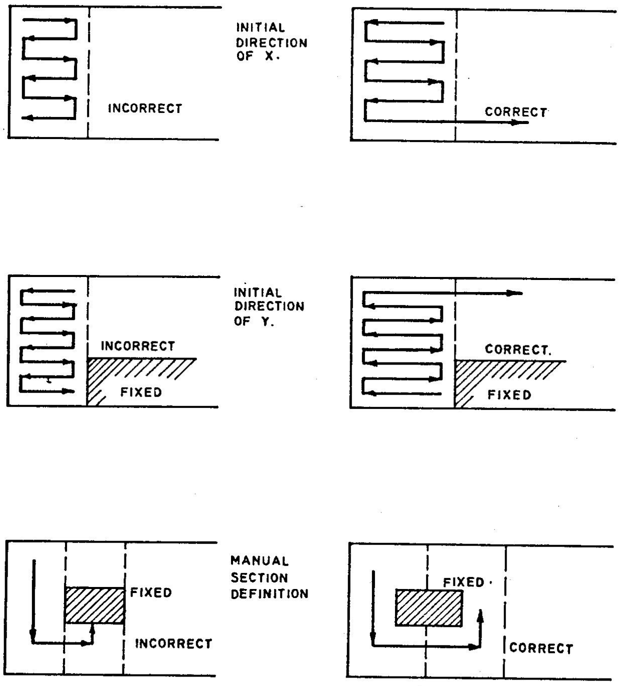

FIGURE 5: THREE MEASURES TO ASSURE CONTINUOUS ALLOCATION.

(b) are partly automated so that, once the principal direction has been defined (externally) for the first section, the program will find a continuous allocation path if such a path exists.

3.2 Evaluation of Layout Plans

Since work centres are two-dimensional structures defined in the same plane, it is necessary to represent them by points in order to measure distances between work centres, d_{ij}. CRAFT uses departmental centres of gravity for this purpose, which, to say the least, are no more arbitrary than any other geometric point. This approach was adopted in the simulation model: Centres of gravity are computed from the coordinates of the cells belonging to each work centre. The d_{ij} can be calculated optionally as rectilinear or direct distances.

The evaluation routine computes total materials handling cost, TMC, as a conventional, strictly linear function of flow volumes per time unit, v_{ij}, transport cost per unit volume and distance, c_{ij}, and measured distances, d_{ij},

$$(3.1)\ \text{TMC} = \Sigma_{i=1}^{n}\ \Sigma_{j=1}^{n}\ v_{ij}\, c_{ij} d_{ij},$$

assuming that $v_{ii}=c_{ii} = d_{ii} =0$. In recognition of the possibility of $c_{ij} \neq c_{ji}$, TMC is computed for directional materials flows $v_{ij} \neq v_{ji}$. As pointed out earlier, (3.1) could be modified to allow for nonlinear relationships between c_{ij} and v_{ij},

$$(3.2)\qquad \text{TMC} = \Sigma_{i=1}^{n}\ \Sigma_{j=1}^{n}\ d_{ij} f_{ij}\,(c,v)$$

or for completely nonlinear handling costs,

$$(3.3)\qquad \text{TMC} = \Sigma_{i=1}^{m}\ \Sigma_{j=1}^{n}\ g_{ij}\ (d,c,v)$$

Since TMC is computed only after a layout plan is complete, and **not** with a view to evaluating speculative changes of the plan, any complications introduced by (3.2) or (3.3) are purely technical inasmuch these objective functions require more computer storage space and execution time.

4. Simulation of Allocation Sequences

The order in which work centres are fitted into a ground plan G to form a layout plan L shall be called the **allocation sequence.** Allocation sequences are provided by the simulator, a computerized pseudo-random number generator whose output — random numbers **r** with $0 \leq r \leq 1$ — is converted into either random or baised-random series of work centre identification numbers. Since the objective is **not** to estimate TMC statistics for the population of layout plans but to

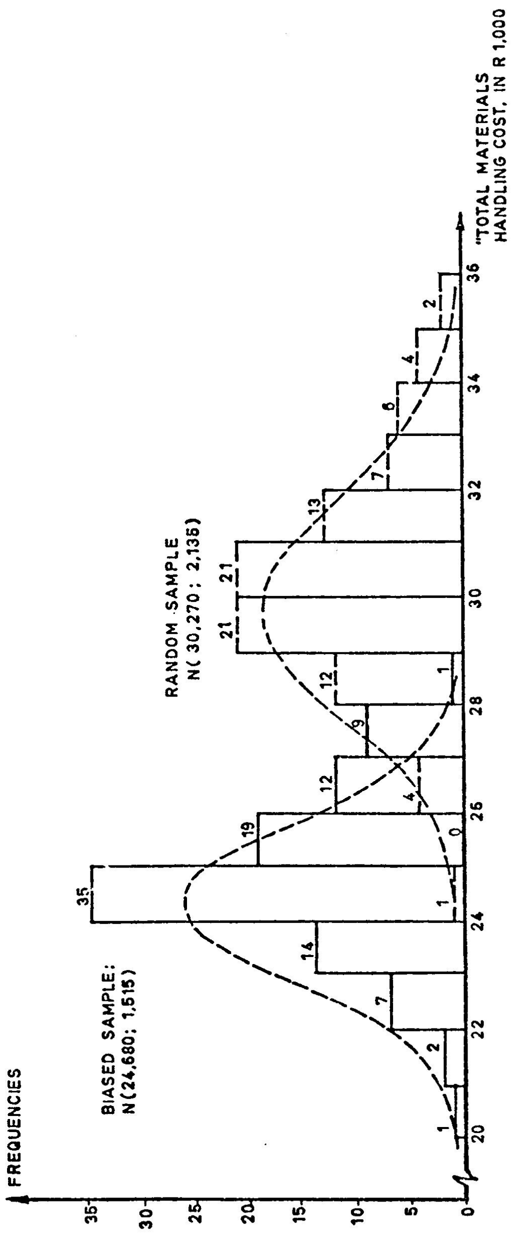

FIGURE 6: HISTOGRAMS AND FITTED NORMAL DISTRIBUTIONS OF A RANDOM AND A DETERMINISTIC-BIASED SAMPLE.

detect arrangements with extremely low TMC, strictly random procedures must be inefficient in comparison with biased-random methods. This speculation is convincingly verified by the sample distributions in Fig. 6, which represent 100 random and biased-random observations each. It is worth noting that in this particular case the best random observation was no better than the mean biased-random equivalent.[1]

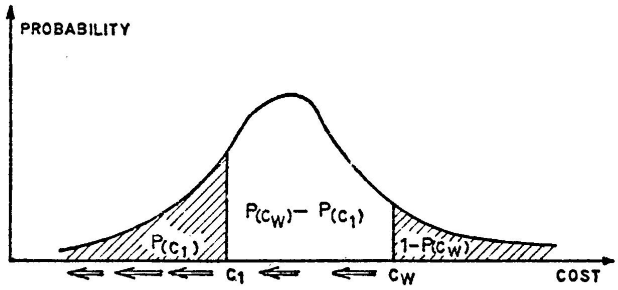

FIGURE 7: DEVELOPMENT OF C_1 AND C_W, AND $P(C_1)$ AND $P(C_W)$

The use of biasing techniques prohibits an application of classical sampling theory formulae to determine run lengths. For this reason, a **dynamic termination criterion** was developed which is entirely feedback oriented and resembles the Las Vegas concept, [1, p.96] It follows the simple notion that if the **w** best results to date are stored, of which TMC_1 is the lowest and TMC_w is the highest period handling cost value, then for any future layout with TMC_j it follows from $TMC_1 < TMC_w$ that

$$(4.1) \quad P(TMC_j \leqq TMC_1) \angle P(TMC_j \leqq TMC_w).$$

Inequality (4.1) indicates that if the set of **w** best scores is continually updated, TMC_w will approach TMC_1 as TMC_1 approaches the unknown lower limit TMC_{min} of which the **model** is capable[2]. In other words, the current lag ($TMC_w - TMC_1$), or the standard deviation, or the coefficient of variation of TMC_i, $i = 1,2\ldots, w$, provides a good indication as to the **stability** and hence — because of (4.1) — the quality of the **w** best solutions at any stage. The computer programs were accordingly designed to accommodate up to $w = 15$ intermediate solutions, and to terminate (optionally) after a predetermind lag, standard deviation, or coefficient of variation has been reached.

5. Summary

A typical layout prepared by the model is given in Fig. 8. Perhaps the most important characteristic of the simulation model is its flexibility with regard to layout construction and evaluation. It distinguishes between variable, linked, and fixed departments, can be adapted to separate work centres where this is deemed desirable, and provides a feasible basis for multi-floor layout planning. Hence, materials handling costs can be calculated, using different types of functions and distances, for arrangements which observe REL chart type constraints imposed on work centre proximities. Alternatively, the flow criterion could be replaced by closeness ratings altogether. The planning flexibility thus implied, is further enhanced by the provision of several nearly equivalent, yet independent, plans in each simulation run, arising as "by-products" of stability testing procedures.

The authors consider as a major finding the fact that even with excessively long runs with the model, using several different simulators, it could not do more than essentially verify the results produced, by the systematic local search method of CRAFT. This suggests that there is little scope for spectacular improvements purely in terms of a narrow handling cost criterion, over what has been accomplished by existing techniques. It therefore enhances the adequacy of visible trends towards broader objectives in computer-aided layout planning, [5,7,8,9,10,13,14].

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[1] For a detailed discussion of biasing techniques see [14].

[2] If the model were free of any idiosyncrasies, this lower limit would be the optimal solution to the problem at hand. Unfortunately, simulators — and especially baised simulators — are never in a rigid sense free of idiosyncrasies.

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