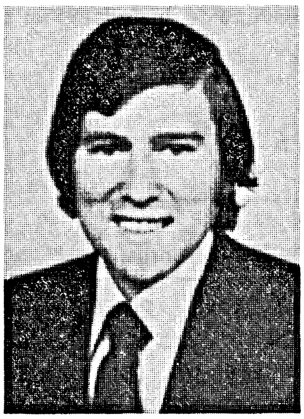
SCHEDULING MULTI-MODEL PRODUCTION LINES



Twee skeduleringsalgoritmes vir multimodelvervaardiging op 'n gegangde produksielyn word voorgestel. Die eerste skeduleringsalgoritme het as besluitbasis die aantal eenhede wat van elke modeltipe in voorraad is op bepaalde tydstippe gedurende die vervaardigingsperiode. Die tydstippe is aan die einde van die vervaardiging van elke produksielot.

Die tweede skeduleringsalgoritme is geskoei op Samuel Eilon se teorie waar die skedule sodanig opgestel word dat daar altyd voorraad op hande is van elke modeltipe wat vervaardig word. 'n Aanvaarbare produksieveldwydte vir die lotgrootte van elke modeltipe word voorgeskryf.

'n Vergelyking tussen die twee skeduleringsalgoritmes word getref.

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INTRODUCTION

Research on the development of an algorithm for the total optimisation of multi-model production lines has proved that line balancing and scheduling can be performed independently of one another when the optimum cycle time and batch sizes are known. This characteristic is an important aspect of the proposed multimodel production process, because line balancing is a time consuming and expensive exercise and the elimination thereof as a prerequisite for scheduling is of great economic benefit to the enterprise.

Two scheduling algorithms for multi-model production on a paced assembly line are proposed.

THE FIRST SCHEDULING ALGORITHM

A scheduling algorithm based on the number of units of each model type in stock at a certain point of time during manufacturing, is developed. The point of time referred to is at the end of the manufacture of each production lot.

It is accepted that a lot of any model type may be next in turn for manufacturing. A lot of each model type is chosen in turn, and the total provision for the scheduling period is calculated by adding the particular lot size to the completed production quantity. From this the scheduling moment is calculated at the end of the particular production lot. The inventory situation of each model type at the particular moment in time, is then calculated. With the inventory situation known, the deviation of the average inventory of all the model types is calculated. The model type lot which yields a minimum deviation is chosen for manufacturing.

A production schedule determined according to the optimization algorithm, is drawn up with reference to the particular optimum cycle time of the production line and optimum lot sizes.

$$\text{optimum cycle time} = 7{,}32 \text{ minutes}$$

$$Q_{e,1} = 30 \text{ units}$$

$$Q_{e,2} = 30 \text{ units}$$

$$Q_{e,3} = 34 \text{ units}$$

Suppose that the schedule period runs into three months, that is 66 working days of 450 minutes each. The period which is lost between the manufacturing of one lot size and the next is five minutes, and unforeseen business idleness amounts to approximately 30 minutes per day. A cycle time of 7,32 minutes yields a production rate of 0,1366 units per minute. Suppose that the actual production time amounts to 450 minutes minus (30 A 3 B 5) minutes C 405 minutes.

The demand for the model types is as follows:

$$a_{c,1} = 0{,}0556 \times 450 = 25 \text{ units/day}$$

$$a_{c,2} = 0{,}0444 \times 450 = 20 \text{ units/day}$$

$$a_{c,3} = 0{,}0222 \times 450 = \underline{10 \text{ units/day}}$$

$$\text{TOTAL} \quad 55 \text{ units/day}$$

The production per day = 0,1366 x 405 = 55 units/day.

The following number of model types will be required for the three month period for which the schedule is drawn up:

$$\text{model type 1} = 25 \times 66 = 1\,650$$

$$\text{model type 2} = 20 \times 66 = 1\,320$$

$$\text{model type 3} = 10 \times 66 = \underline{\;660}$$

$$\text{TOTAL} \quad 3\,630 \text{ units}$$

The total number of units of a model type which is manufactured, follows from the equation:

$$y_h \cdot Q_{e,h} = \text{total number of units of model type (h)}$$

$$\text{where } h = 1, 2 \ldots\ldots, H$$

The number of optimum lot sizes which must be manufactured of each one, is as follows:

$$y_1 = \frac{1\,650}{30} = 55 \text{ lots}$$

$$y_2 = \frac{1320}{30} = 44 \text{ lots}$$

$$y_3 = \frac{660}{34} = \underline{20} \text{ lots}$$

119 lots total

The reliability of these figures can be tested as follows:

$$\text{Total production time} = \frac{\bar{C}_{opt}}{450} \times 3\,630$$

$$= 59 \text{ days}$$

The preparation time and unforeseen business idleness wil be as follows:

$$\frac{119 \text{ lot sizes} \times 5 \text{ minutes}}{450} = 1{,}3$$

$$\underline{30 \text{ minutes} \times 66} = \underline{4{,}4}$$

TOTAL IDLE TIME say 6 days

The total schedule time = 59 + 6 = 65 days, which, for all practical purposes, is equivalent to the schedule period of 66 days.

The 119 lots which are manufactured during the schedule period can be arranged in any alternative sequence, so long as the daily demand for each model type is satisfied in the best possible way. Table 1 serves as an example of how the model type lots can be scheduled.

The first column indicates the number of the lot; the second column the model type and lot size which is manufactured $(Q_{e,h})$;

the third column the total number of units manufactured $\sum_{h=1}^{3} Q_{e,h}$;

the fourth column the moment of the schedule

$$\bar{C}_{opt.} \cdot \sum_{h=1}^{3} y_h Q_{e,h} \cdot 1{,}11;$$

the fifth column the market demand for the model types at that moment (ac,h. moment of schedule); the sixth column the total provision at that moment $(y_h \cdot Q_{e,h})$; and the last column indicates whether there is a shortage or a surplus of each model type.

The correction factor of 1,11 arises from the fact that the daily production time takes up 405 minutes, while the market demand is determined over 450 minutes. The correction factor is therefore:

$$\frac{450}{405} = 1{,}11 = F = \text{multiplication factor}$$

The schedule is drawn up in such a way that inventory shortages and surplusses of model types balance each other.

Development of the step which rounds off the scheduling algorithm

To carry out scheduling of the model type-lot sizes in a more scientific way, the following step has been developed and added to the scheduling algorithm. The step flows directly from the result which was obtained with the scheduling in table 1.

The last column in table 1 indicates the inventory situation of the various model types after the particular lot size has been manufactured. This gave rise to the idea of developing a mathematical step whereby shortages and surplusses of the model types could be minimized in relation to one another.

Suppose that the inventory situation after the manufacture of a lot of any model type is indicated by (Vh), where h = 1,2, ..., H.

Choose the first model type-lot size in such a way that

$$\left|Z - (V_1)\right| + \left|Z - (V_2)\right| + \; + \left|Z - (V_h)\right| + \; + \left|Z - (V_H)\right| = \text{minimum}$$

$$\text{where } Z = \frac{V_1 + V_2 + \; + V_h + \; + V_H}{H}$$

after steps (a) to (f) as indicated in table 1 have been performed for each model type.

Choose the second model type-lot size by performing the exact same step as indicated above.

To simplify the calculation, the previous model type which was scheduled is not considered again in the next apportionment. Table 2 serves only as an illustration of the method. The programmed scheduling algorithm considers all the model types step by step, including the model type which has just been apportioned. This result is reflected in table 3, and indicates that the same model type is sometimes scheduled immediately after another.

The complete scheduling algorithm is then as follows:

1. Choose a model type-lot size where $(Q_{e,h})$ where $h = 1, 2 \ldots\ldots, H$ (1)
2. Suppose that the schedule is complete to moment t. Now, to determine which model must be scheduled next, the following steps are carried out:

Let y_h be manufactured to type h;

$h = 1, \ldots\ldots, H.$

For $J = 1, \ldots\ldots, H$

(a) Let $y'_h = \begin{cases} y_h & h \neq j \\ y_{h+1} & h = j \end{cases}$ (2)

(b) Determine $\sum_{h=1}^{H} y'_h \cdot Q_{e,h}$ (3)

Determine $T_{pj} = \bar{C}_{opt} \left(\sum_{h=1}^{H} y_h \cdot Q_{e,h} \right) \cdot F$ (4)

$$\text{where } F = \frac{\text{total time per shift}}{\text{actual production time per shift}}$$

(c) Determine the market demand for each model type at the particular moment in the schedule period $= a_{c,h} \cdot T_{pj}$ (5)

TABLE 1

THE PRODUCTION SCHEDULE AS COMPILED FOR THE FIRST EIGHT WORKING DAYS OF THE SCHEDULE PERIOD

Lot size number	(a) Model type lot size $Q_{e,h}$			(b) Total $(\sum_{h=1}^{3} Y_h Q_{e,h})$	(c) Schedule moment $(\bar{C}_{opt} \Sigma Y_h Q_{e,h} \cdot 1,11)$		(d) Market demand $(a_{c,h} \cdot \text{moment})$			(e) Total provision $(Y_h \cdot Q_{e,h})$			(f) Stock (V_h) (e) - (d)		
	1	2	3		minutes	days	1	2	3	1	2	3	1	2	3
1	30	0	0	30	243,75	0,54	13	11	5	30	0	0	17	-11	-5
2	0	30	0	60	487,50	1,08	27	22	11	30	30	0	3	8	-11
3	0	0	34	94	763,75	1,70	42	34	17	30	30	34	-12	-4	17
4	30	0	0	124	1007,50	2,24	56	45	22	60	30	34	4	-15	12
5	0	30	0	154	1251,23	2,78	69	55	28	60	60	34	-9	5	6
6	30	0	0	184	1495,00	3,32	83	66	33	90	60	34	7	-6	1
7	0	30	0	214	1738,75	3,86	97	77	39	90	90	34	-7	13	-5
8	0	0	34	248	2015,00	4,48	112	89	45	90	90	68	-22	1	23
9	30	0	0	278	2258,75	5,02	125	100	50	120	90	68	-5	-10	18
10	0	30	0	308	2502,50	5,56	139	111	56	120	120	68	-19	9	12
11	30	0	0	338	2746,25	6,10	152	122	61	150	120	68	-2	-2	7
12	0	30	0	368	2990,00	6,64	166	133	66	150	150	68	-16	17	2
13	30	0	0	398	3233,74	7,19	179	143	72	180	150	68	1	7	4
14	0	0	34	432	3470,00	7,71	194	156	78	180	150	102	-4	-6	24
15	30	0	0	462	3753,74	8,34	208	167	83	210	150	102	2	-17	19

TABLE 2

THE REVISED PRODUCTION SCHEDULE AS COMPILED FOR THE FIRST NINE MODEL TYPE-LOT SIZES AFTER THE OPTIMIZING STEP HAS BEEN ADDED TO THE SCHEDULING ALGORITHM

Lot number	(a) Model type-lot size $Q_{e,h}$			(b) Total $\sum_{h=1}^{H} y_h Q_{e,h}$	(c) Schedule moment $\bar{C}_{opt} \Sigma y_h Q_{e,h} \cdot F$		(d) Market demand $a_{c,h} \cdot \text{moment}$			(e) Total provision $y_h \cdot Q_{e,h}$			(f) Stock (e)-(d) V_h			(g) $\sum_{h=1}^{H} \lvert Z - (V_h) \rvert$	(h) Choice of model type
	1	2	3		minutes	days	1	2	3	1	2	3	1	2	3		
1	30	0	0	30	243,75	0,54	13	11	5	30	0	0	17	-11	-5	33,3	1
	0	30	0	30	243,75	0,54	13	11	5	0	30	0	-13	19	-5	37,3	
	0	0	34	34	276,25	0,61	15	12	6	0	0	34	-15	-12	28	55,3	
2	0	30	0	60	487,50	1,08	27	22	11	30	30	0	3	8	-11	22,0	2
	0	0	34	64	520,00	1,16	29	23	12	30	0	34	1	-23	22	46,0	
3	0	0	34	94	763,75	1,70	42	34	17	30	30	34	-12	-4	17	33,3	3
	30	0	0	90	731,25	1,63	40	32	16	60	30	0	20	-2	-16	38,6	
4	30	0	0	124	1007,50	2,24	56	45	22	60	30	34	4	-15	12	30,6	1
	0	30	0	124	1007,50	2,24	56	45	22	30	60	34	-26	15	12	53,6	
5	0	30	0	154	1251,25	2,78	69	56	28	60	60	34	-9	4	6	18,6	2
	0	0	34	158	1283,75	2,85	71	57	29	60	30	68	-11	-27	39	77,3	
6	30	0	0	184	1495,00	3,32	83	66	33	90	60	34	7	-6	1	13,3	1
	0	30	0	184	1495,00	3,32	83	66	33	60	90	34	-23	24	1	47,3	
7	0	30	0	214	1738,75	3,86	97	77	39	90	90	34	-7	13	-5	25,3	2
	0	0	34	218	1771,25	3,94	98	79	39	90	60	68	-8	-19	29	56,6	
8	0	0	34	248	2015,00	4,48	112	89	45	90	90	68	-22	1	23	45,3	1
	30	0	0	244	1982,50	4,41	110	88	44	120	90	34	10	2	-10	21,3	
9	0	0	34	278	2258,75	5,02	125	100	50	120	90	68	-5	-10	16	31,3	3
	0	30	0	274	2226,30	4,95	124	99	49	120	120	34	-4	21	-15	40,6	

(d) Determine the inventory level (V_h) of each model type at the particular moment in the schedule period.

$$V_h = (Y_h \cdot Q_{e,h}) - (a_{c,h} \cdot T_{pj}) \quad (6)$$

(e) Determine $P_j = \sum_{h=1}^{H} \left| Z - (V_h) \right|$ (7)

where $Z = \dfrac{\sum_{h=1}^{H} V_h}{H}$

3. The model type which yields a minimum value for P_j in (7), is included in the production schedule.

To demonstrate the new step which is added to the scheduling algorithm, the determination of the first nine model type lot-sizes of the schedule as reproduced in table 1, is repeated. The result of the calculations is reproduced in table 2 and clearly indicates the validity of the new step which has been developed.

Proof is thereby furnished in table 1 that a mistake was made in the scheduling when model type 3 was accepted as lot size 8 instead of model type 1. The development of this step in the scheduling algorithm can be regarded as an important phase in this research, especially when one considers that the step ensures that there is always a balance between the units in stock of each model type.

Table 2 illustrates the efficiency of the scheduling algorithm.Column (a) indicates the model type-lot sizes.

Columns (b) to (f) are self explanatory, while column (g) indicates the deviations of the particular production lots of the alternative model types. Column (h) indicates which model type-lot yields minimum deviation, and it represents the sequencing of the model type-lots on the production line, that is the schedule. Scheduling in accordance with the method first proposed thus embraces the sequencing of production lots of the various model types which are manufactured on the paced production line. The multi-model algorithm functions efficiently and generates a schedule which ensures that there is a good balance between the number of available inventory units of the model types, while at the same time satisfying the market needs.

The scheduling algorithm is programmed for application on the IBM 360 model 50 computer. To test the programme the compiling of a ten-day schedule for the manufacture of the three model types of the enterprise is performed on the digital computer. This result appears in table 3 and yields the same schedule as table 2.

Inspection of the inventory situation of the model types during manufacturing shows that a model type is out of stock at one or other stage (see table 4 for an extract from the schedule). This is the only disadvantage in the first scheduling method, but it has no serious implications. The out-of-stock situation only lasts for a few hours in the case of model types 1 and 2, whereas the worst case occurs with model type 3 which is out of stock for one whole day. However, the fact that no provision is made for buffer stock must be taken into account. As we have already stated, it is unnecessary to make provision for a buffer stock owing to the nature of the carrying out of orders at the factory.

For practical purposes this condition is acceptable and the short period for which a model type is out of stock has no influence worth mentioning on the optimality of production. This statement is made in light of the fact that the dispatch of orders at the factory can even take several days to be disposed of administratively, before physical dispatch occurs. Simply stated this means that a motor vehicle is not a commodity which is sold over the counter at the factory, and it is not necessary for stock to be available at all times.

The great advantage of the first scheduling algorithm is the manufacture of precise optimum lot sizes as specified. Because the sizes of the production lots are precisely the same as the specified optimum lot sizes, it is unnecessary to specify a production range and no increase in variable cost occurs. The inventory situation is controlled by the sequencing of the various model type-lots in the schedule. Because the cycle time of the production line is tuned in to the market demand, with the necessary allowances for unforeseen business idleness, market needs during the schedule period are satisfied exactly.

THE SECOND SCHEDULING ALGORITHM

The first scheduling algorithm which was proposed can be compared with a second method, which casts Eilons[2] theory in a different mould, and where the schedule is drawn up in such a way that there is always stock on hand of each model type which is manufactured. The quantities of a model type which are manufactured at a time in this scheduling algorithm, deviate from the prescribed economic lot sizes and it is necessary for an acceptable production range to be prescribed for the lot sizes of each model type.

The following mathematical equations are formulated to describe the second multi-model scheduling.

The damand period Tc,h of each model type is as follows:

$$\sum_{h=1}^{H} \left(\max_i T_{s,i} + \sum_{i=1}^{n} T_{p,i} \right) h + \text{unforeseen idleness}$$

$$= \sum_{h=1}^{H} \left(\max_i T_{s,i} + Q.\bar{C}_{opt.} \right) h + \text{unforeseen idleness} \quad (8)$$

The above equation makes provision for a model type being manufactured only once during the total demand period.

$$\text{And } T_{c,1} = T_{c,2} = T_{c,h} \ldots = T_{c,H} = \left(\frac{Q}{a_c} \right) h \quad (9)$$

TABLE . 3

SCHEDULING RESULT OBTAINED BY APPLYING THE PROGRAMMED SCHEDULING ALGORITHM

LOT NO.	LOT SIZE QE (H)			TOT. QE (H)	SCHEDULE MOMENT		MARKET DEMAND			TOT. PROVISION			STOCK			ST.DEV.	MIN H
					MINUTES	DAYS	1	2	3	1	2	3	1	2	3		
1	30	0	0	30	243,51	0,54	13	10	5	30	0	0	17	-10	-5	32,67	
1	0	30	0	30	243,51	0,54	13	10	5	0	30	0	-13	20	-5	38,67	
1	0	0	34	34	275,97	0,61	15	12	6	0	0	34	-15	-12	28	55,33	1
2	30	0	0	60	487,01	1,08	27	21	10	60	0	0	33	-21	-10	64,67	
2	0	30	0	60	487,01	1,08	27	21	10	30	30	0	3	9	-10	21,33	
2	0	0	34	64	519,48	1,15	28	23	11	30	0	34	2	-23	23	47,33	2
3	30	0	0	90	730,52	1,62	40	32	16	60	30	0	20	-2	-16	38,67	
3	0	30	0	90	730,52	1,62	40	32	16	30	60	0	-10	28	-16	54,67	
3	0	0	34	94	762,99	1,70	42	33	16	30	30	34	-12	-3	18	34,00	3
4	30	0	0	124	1006,49	2,24	55	44	22	60	30	34	5	-14	12	30,00	
4	0	30	0	124	1006,49	2,24	55	44	22	30	60	34	-25	16	12	52,00	
4	0	0	34	128	1038,96	2,31	57	46	23	30	30	68	-27	-16	45	88,67	1
5	30	0	0	154	1250,00	2,78	69	55	27	90	30	34	21	-25	7	52,00	
5	0	30	0	154	1250,00	2,78	69	55	27	60	60	34	-9	5	7	20,00	
5	0	0	34	158	1282,47	2,85	71	56	28	60	30	68	-11	-26	40	78,00	2
6	30	0	0	184	1493,50	3,32	83	66	33	90	60	34	7	-6	1	13,33	
6	0	30	0	184	1493,50	3,32	83	66	33	60	90	34	-23	24	1	47,33	
6	0	0	34	188	1525,97	3,39	84	67	33	60	60	68	-24	-7	35	67,33	1
7	30	0	0	214	1737,01	3,86	96	77	38	120	60	34	24	-17	-4	46,00	
7	0	30	0	214	1737,01	3,86	96	77	38	90	90	34	-6	13	-4	24,00	
7	0	0	34	218	1769,48	3,93	98	78	39	90	60	68	-8	-18	29	56,00	2
8	30	0	0	244	1980,52	4,40	110	87	43	120	90	34	10	3	-9	20,67	
8	0	30	0	244	1980,52	4,40	110	87	43	90	120	34	-20	33	-9	63,33	
8	0	0	34	248	2012,98	4,47	111	89	44	90	90	68	-21	1	24	45,33	1
9	30	0	0	274	2224,02	4,94	123	98	49	150	90	34	27	-8	-15	51,33	
9	0	30	0	274	2224,02	4,94	123	98	49	120	120	34	-3	22	-15	41,33	
9	0	0	34	278	2256,49	5,01	125	100	50	120	90	68	-5	-10	18	34,00	3
10	30	0	0	308	2500,00	5,56	138	110	55	150	90	68	12	-20	13	43,33	
10	0	30	0	308	2500,00	5,56	138	110	55	120	120	68	-18	10	13	39,33	
10	0	0	34	312	2532,46	5,63	140	112	56	120	90	102	-20	-22	46	89,33	2
11	30	0	0	338	2743,50	6,10	152	121	60	150	120	68	-2	-1	8	12,67	
11	0	30	0	338	2743,50	6,10	152	121	60	120	150	68	-32	29	8	67,33	
11	0	0	34	342	2775,97	6,17	154	123	61	120	120	102	-34	-3	41	79,33	1
12	30	0	0	368	2987,01	6,64	166	132	66	180	120	68	14	-12	2	26,67	
12	0	30	0	368	2987,01	6,64	166	132	66	150	150	68	-16	18	2	34,67	
12	0	0	34	372	3019,48	6,71	167	134	67	150	120	102	-17	-14	35	67,33	1
13	30	0	0	398	3230,52	7,18	179	143	71	210	120	68	31	-23	-3	58,67	
13	0	30	0	398	3230,52	7,18	179	143	71	180	150	68	1	7	-3	10,67	
13	0	0	34	402	3262,98	7,25	181	144	72	180	120	102	-1	-24	30	56,67	2
14	30	0	0	428	3474,02	7,72	193	154	77	210	150	68	17	-4	-9	31,33	
14	0	30	0	428	3474,02	7,72	193	154	77	180	180	68	-13	26	-9	49,33	
14	0	0	34	432	3506,49	7,79	194	155	77	180	150	102	-14	-5	25	46,00	1

*** Owing to its large volume the complete table is not reproduced here.**

TABLE 4 REPRESENTATION OF THE INVENTORY SITUATIONS OF MODEL TYPES AFTER THE MANUFACTURE OF A PARTICULAR MODEL TYPE WHICH IS SCHEDULED

Model type which is scheduled	Inventory situation of model types: Model type 1	Model type 2	Model type 3	Schedule moment in days
1	12	-7	6	19,91
2	-3	12	0	20,45
1	15	1	-5	21,00
2	2	20	-11	21,54
3	-14	8	17	22,15
1	3	-3	12	22,69
2	-11	16	6	23,23
1	6	6	1	23,77
1	22	-5	-4	24,31
2	9	14	-10	24,86
3	-7	2	18	25,77
1	10	-9	13	26,01
2	-4	10	7	26,55
1	13	-1	2	27,09

If $a_h = a_{c,h} / a_{c,1}$ $h = 2, 3, \ldots, H$ (10)

then $Q_h = a_h \cdot Q_1$ (11)

The equations as indicated above ensure that there is always stock on hand of each model type, and that the building up of excessive stock is eliminated. The question which now arises is whether lot sizes must be prescribed for the various model types to ensure maximum efficiency of the total schedule. The rate of return of the total schedule is used as the point of departure, and the relationship of profit to cost of production per unit of time is maximized. From Eilon's theory it follows that the rate of return for a single model type during the demand period, is as follows:

$$D = \frac{1}{T_c} \cdot \frac{QY' - QY}{QY}$$

For the multi-model case and conditions as described above, the expression is

$$D_{\Sigma h} = \sum_{h=1}^{H} \frac{(Q_h \cdot Y'_h - Q_h \cdot Y_h)}{\sum_{h=1}^{H} Q_h \cdot Y_h} \cdot \frac{1}{T_c} \tag{12}$$

substitute $T_c = \frac{Q_1}{a_{c,1}}$ and $Q_h = \alpha_h \cdot Q_1$

$$\therefore D_h = a_{c,1} \frac{\sum_{h=1}^{H} \alpha_h \cdot Y'_h}{\sum_{h=1}^{H} \alpha_h \cdot Y_h \cdot Q_1} - \frac{1}{Q_1} \tag{13}$$

The expression is maximized by means of differentiation with reference to Q_1 and by equalling it to zero.

$$\frac{d D_{\Sigma h}}{d Q_1} = 0$$

$$\frac{\sum_{h=1}^{H} \alpha_h \cdot Y'_h}{Q_1^2 \cdot \left(\sum_{h=1}^{H} \alpha_h \cdot Y_h\right)^2} \cdot \frac{d}{d Q_1} \left(\sum_{h=1}^{H} \alpha_h \cdot Y_h \cdot Q_1\right) = \frac{1}{Q_1^2}$$

$$\therefore \sum_{h=1}^{H} \alpha_h \cdot Y'_h \cdot \frac{d}{dQ_1} \left(\sum_{h=1}^{H} \alpha_h \cdot Y_h \cdot Q_1\right) = \left(\sum_{h=1}^{H} \alpha_h \cdot Y_h\right)^2 \tag{14}$$

but

$$\alpha_h \cdot Y_h \cdot Q_1 = \alpha_h \cdot Q_1 \left(\sum_{i=1}^{n} c_{i,h} + \frac{\sum_{i=1}^{n} S_{i,h}}{\alpha_h \cdot Q_1} + K_h \cdot \alpha_h \cdot Q_1\right) \tag{15}$$

and

$$\frac{d}{dQ_1} \left(\sum_{h=1}^{H} \alpha_h \cdot Y_h \cdot Q_1\right) = \sum_{n=1}^{H} \left(\alpha_h \cdot \sum_{i=1}^{n} c_{i,h}\right) + 2Q_1 \cdot \sum_{h=1}^{H} K_h \cdot \alpha_h^2 \tag{B 16}$$

also

$$\left(\sum_{h=1}^{H} \alpha_h \cdot Y_h\right)^2 = \left[\sum_{h=1}^{H} \left(\alpha_h \cdot \sum_{i=1}^{n} c_{i,h} + \frac{\sum_{i=1}^{n} S_{i,h}}{Q_1} + K_h \cdot \alpha_h^2 \cdot Q_1\right)\right]^2$$

$$= \sum_{h=1}^{H} \left(\alpha_h \cdot \sum_{i=1}^{n} c_{i,h}\right) + \left(\frac{\sum_{h=1}^{H} \sum_{n=1}^{n} C_{i,h}}{Q_1}\right)^2$$

$$+ Q_1 \left(\sum_{h=1}^{H} K_h \cdot \alpha_h^2\right)^2 + 2 \cdot \frac{\left(\sum_{h=1}^{H} \alpha_h \cdot \sum_{i=1}^{n} c_{i,h}\right) \left(\sum_{h=1}^{H} \sum_{i=1}^{n} c_{i,h}\right)}{Q_1}$$

$$+ 2 \left(\sum_{h=1}^{H} \sum_{i=1}^{n} s_{i,h} \cdot \sum_{h=1}^{H} K_h \cdot \alpha_h^2\right)$$

$$+ 2Q_1 \cdot \sum_{h=1}^{H} \left(\alpha_h \cdot \sum_{i=1}^{n} c_{i,h}\right) \left(\sum_{h=1}^{H} K_h \cdot \alpha_h^2\right) \tag{17}$$

Substitution of these values in equation (B 14) and multiplication by Q^2 yields

$$Q_1^4 \left(\sum_{h=1}^{H} K_h \cdot \alpha_h^2\right)^2 - 2Q_1^3 \left(\sum_{h=1}^{H} \alpha_h \cdot Y'_h - \sum_{h=1}^{H} \alpha_h \cdot \sum_{i=1}^{n} c_{i,h}\right) \sum_{h=1}^{H} K_h \cdot \alpha_h^2$$

$$+ Q_1^2 \left[\left(\sum_{h=1}^{H} \alpha_h \cdot \sum_{i=1}^{n} c_{i,h}\right)^2 + 2 \cdot \sum_{h=1}^{H} \sum_{i=1}^{n} s_{i,h} \cdot \sum_{h=1}^{H} K_h \cdot \alpha_h^2 \right.$$

$$\left. - \sum_{h=1}^{H} \left(\alpha_h \cdot Y'_h \cdot \alpha \sum_{h=1}^{H} \alpha_h \cdot \sum_{i=1}^{n} c_{i,h}\right)\right] + 2Q_1 \left(\sum_{h=1}^{H} \alpha_h \cdot \sum_{i=1}^{n} c_{i,h}\right) \sum_{h=1}^{H} \sum_{i=1}^{n} s_{i,h}$$

$$+ \sum_{h=1}^{H} \sum_{i=1}^{n} s_{i,h} = 0 \tag{18}$$

This expression can be simplified by dividing by $\sum_{h=1}^{H} \sum_{i=1}^{n} s_{i,h}$ and by substituting the following equations

$$Q_{M1}^2 = \frac{\sum_{h=1}^{H} \sum_{i=1}^{n} s_{i,h}}{\sum_{h=1}^{H} K_h \cdot \alpha_h^2} \tag{19}$$

$$q_1 = \frac{Q_{E1}}{Q_{M1}} \tag{20}$$

where Q_{M1} = the optimum lot size of model type 1 when the schedule is optimized for maximum returns

and Q_{E1} = the optimum lot size of model type 1 for maximum rate of return for the schedule

$$\therefore q_1^4 - 2 \frac{\sum_{h=1}^{H} c_h \cdot Y'_h - \sum_{h=1}^{H} \alpha_h \cdot \sum_{i=1}^{n} c_{i,h}}{\sum_{h=1}^{H} \sum_{i=1}^{n} s_{i,h} / Q_{M1}} q_1^3$$

$$+ \left(2 - \frac{\sum_{h=1}^{H} \alpha_h \cdot Y'_h - \sum_{h=1}^{H} \alpha_h \cdot \sum_{i=1}^{n} c_{i,h}}{\sum_{h=1}^{H} \sum_{i=1}^{n} s_{i,h} / Q_{M1}} \cdot \frac{\sum_{h=1}^{H} \alpha_h \cdot \sum_{i=1}^{n} c_{i,h}}{\sum_{h=1}^{H} \sum_{i=1}^{n} s_{i,h} / Q_{M1}}\right) q_1^2$$

$$+ 2 \frac{\sum_{h=1}^{H} \alpha_h \cdot \sum_{i=1}^{n} c_{i,h}}{\sum_{h=1}^{H} \sum_{i=1}^{n} s_{i,h} / Q_{M1}} \cdot q_1 + 1 = 0 \tag{21}$$

Substitute the following non-dimensional relationships

$$P_1^1 = \frac{\sum_{h=1}^{H} \alpha_h \cdot Y'_h - \sum_{h=1}^{H} \alpha_h \cdot \sum_{i=1}^{n} c_{i,h}}{2 \cdot \sum_{h=1}^{H} \sum_{i=1}^{n} s_{i,h} / Q_{M1}} \tag{22}$$

$$U_1 = \frac{\sum_{h=1}^{H} \alpha_h \cdot \sum_{i=1}^{n} c_{i,h}}{\sum_{h=1}^{H} \sum_{i=1}^{n} s_{i,h} / Q_{M1}} \tag{23}$$

which results in the following expression

$$q_1^4 - 4 P_1^1 q_1^3 + 2(1 - U_1 P_1^1) q_1^2 + 2U_1 q_1 + 1 = 0 \tag{24}$$

This equation can be solved by means of the computer programme which has been developed for this purpose, or by means of the method of approximation proposed by Eilon [3]

The solution for the production lot sizes of the three model types is as follows (see table 5):

$$Q_{E1} = 44$$

$$Q_{E2} = 35$$

$$Q_{E2} = 18$$

From the results of a previous publication it is clear that a 5 per cent increase in the variable cost element has no influence on the optimality of production[4] On the basis of this finding a variable cost factor of p = 1,10 can be allowed in determining the lot size boundaries. The result is indicated in table 6. From the results it is clear that QE1 is just outside the 5 per cent limit and that QE2 is comfortably within the 5 per cent limit. For model type 3,QE3 is on the 18 per cent limit and thus falls outside the general prescribed maximum limit for an increase in variable cost.

The poorest seller of all the models is model type 3 however, but the enterprise can accept the 18 per cent, especially seeing that the schedule is so favourable for the other two model types. To test the applicability of the new schedule, it is subjected to the cycle test.

TABLE 5

CALCULATION OF THE PRODUCTION LOT SIZES ACCORDING TO THE SECOND SCHEDULING ALGORITHM – DATA AS OBTAINED FROM TABLES 1 AND 2 OF A PREVIOUS PUBLICATION[4])

$$\alpha_1 = \frac{0,0555}{0,0555} = 1$$

$$\alpha_2 = \frac{0,0444}{0,0555} = 0,8$$

$$\alpha_3 = \frac{0,0222}{0,0555} = 0,4$$

$$Y'_1 = 1200; \quad Y'_2 = 1250; \quad Y'_3 = 1350$$

$$\sum_{h=1}^{H} \alpha_h . Y'_h = 1200 + (0,8 \times 1250) + (0,4 \times 1350) = 2740$$

$$\sum_{h=1}^{H} \alpha_h . \sum_{i=1}^{n} c_{i,h} = 1100 + (0,8 \times 1150) + (0,4 \times 1250) = 2520$$

$$\sum_{h=1}^{H} \sum_{i=1}^{n} s_{i,h} = 1500 + 1500 + 1700 = 4700$$

$$\sum_{h=1}^{H} (\alpha^2 . K)_h = (0,19 \times 1) + (0,25 \times 0,64) + (0,5 \times 0,16) = 0,43$$

$$Q_{M1} = \sqrt{\frac{4700}{0,43}} = 104,55 \text{ say } 105$$

$$Q_{E1} = 105 \times q_1$$

$$P'_1 = \frac{2740 - 2520}{2 \times \frac{4700}{105}} = 2,46$$

$$q_1 = 0,42$$

$$Q_{E1} = 105 \times 0,42 = 44$$

$$Q_{E2} = 44 \times 0,8 = 35$$

$$Q_{E3} = 44 \times 0,4 = 18$$

$$Q_{E1} / a_{c,1} = 44/0,0556 = 790 \text{ minutes}$$

$$Q_{E2} / a_{c,2} = 35/0,0444 = 790 \text{ minutes}$$

$$Q_{E3} / a_{c,3} = 18/0,0222 = 810 \text{ minutes}$$

From equation 8 the demand period is

$$\sum_{h=1}^{H} \left(\max_i T_{s,i} + Q.\bar{C}_{opt} \right) h + \text{unforeseen idleness}$$

$$= 15 + (97 \times 7,32) + (\frac{790}{450} \times 30)$$

$$= 15 + 710 + 53 = 778 \text{ min.}$$

The demand period almost coincides with the production period and further investigation can be instituted to determine whether a better schedule can be found. The lot sizes of model types 1 and 2 can first of all be increased by one and secondly, the lot sizes of all the model types can be reduced by one.

The first alternative yields demand periods which are exactly equal for all three model types.

Model type 1: 45/0,0556 = 810 min.
Model type 2: 46/0,0444 = 810 min.
Model type 3: 18/0,0222 = 810 min.
Production time: 15 + (99 x 7,32) + 55 = 795 min.

The second alternative yields the following results:

Model type 1: 43/0,0556 = 775 min.
Model type 2: 34/0,0444 = 767 min.
Model type 3: 17/0,0222 = 765 min.
Production time: 15 + (94 x 7,32) + 51 = 755 min.

When the alternatives are compared with the original solution the conclusion can then be drawn that there is little to choose between the original solution and the first alternative. Because the original solution does not deviate from the prescribed lot sizes it must be regarded as the more acceptable one, especially since model type 3 is the less important seller. If $17\frac{1}{2}$ units of model type 3 could be manufactured, the demand price thereof would also be 790 minutes. However, it is impossible to manufacture half a unit and the solution is to manufacture 17 and 18 alternatively per lot of model type 3.

The practical implication of this solution is that the production cycle is 1,4 per cent faster than the demand period of the model types. This only represents a slight difference which is so small in practice, when one considers that the difference can easily be absorbed by unforseen idleness.

DISCUSSION OF THE RESULTS

The results indicate that the scheduling algorithm is successful because it satisfies the cycle test as well as the test for the increase in variable cost. The latter is the case when an increase of 18 per cent in variable cost is allowed for model type 3. Model types 1 and 2, the best sellers in the series, are both in a favourable position in that they only experience an increase of approximately 5 per cent in variable cost when lot sizes of 44,35 and 18 are manufactured of the various model types (see table 5).

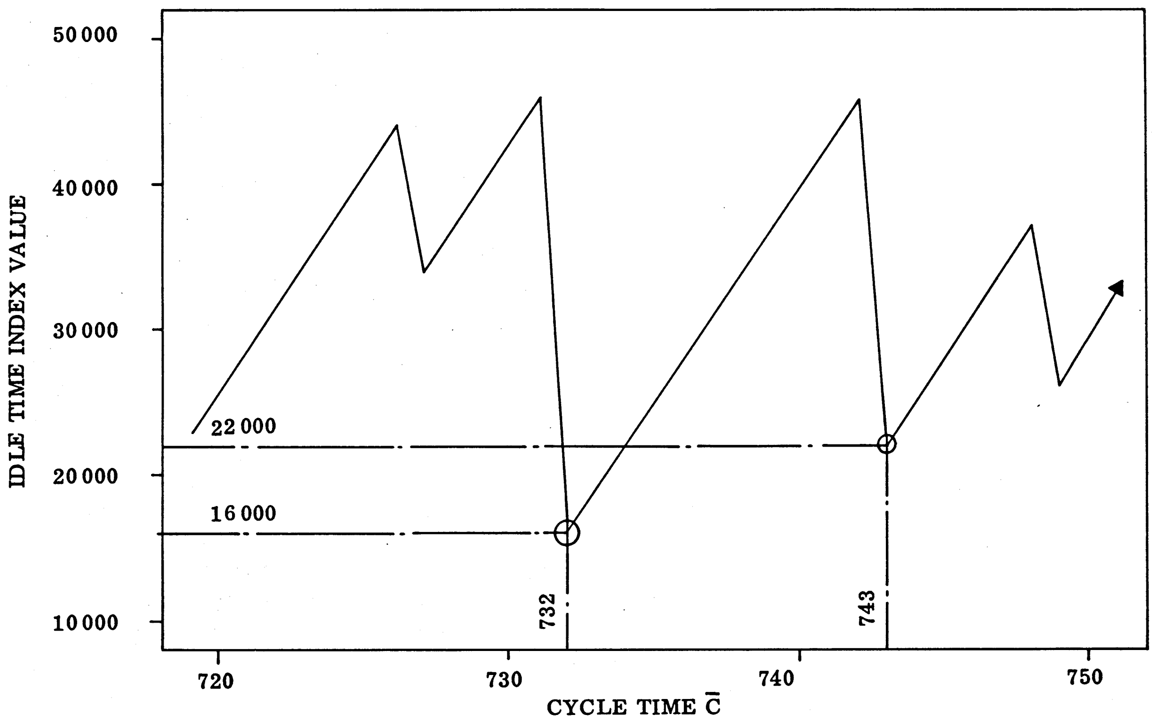
Two alternatives are proposed for the lot sizes which are manufactured. In the first alternative the demand periods of the three model types are exactly equal to one another at 810 minutes whereas the production time is 2 per cent lower. With this alternative 45,36 and 18 units per lot of model types 1, 2 and 3 are manufactured respectively, and it is the only other practical alternative to the original solution. However, the original solution is better in that the demand period is 790 minutes while the production time is 1,4

Lot sizes		p = 1,05		p = 1,10			
Model types	$Q_{e,h}$	$Q_{I,h}$	$Q_{II,h}$	$Q_{I,h}$	$Q_{II,h}$	$Q_{E,h}$	Remark
1	30	22	42	20	47	44	acceptable
2	30	22	42	20	47	35	acceptable
3	34	25	47	22	53	18	acceptable if 18% increase in variable cost is allowed for model type 3

TABLE 6

LOT SIZE BOUNDARIES OF THE MODEL TYPES WHICH ARE MANUFACTURED

FIGURE 7 REPRESENTATION OF THE INCREASE IN IDLE TIME INDEX VALUE WHEN A CYCLE TIME OF 7,43 MINUTES IS USED INSTEAD OF ONE OF 7,32 MINUTES



per cent lower, and it has the added advantage of not deviating from the prescribed lot sizes of 44, 35 and 18 respectively.

It can be argued that the cycle time of the production line must be adjusted to compensate for the difference of 1,4 per cent (or 12 minutes) between the production period and demand period, that is by using a cycle time of 7,43 minutes instead of 7,32 minutes. The implications of such a step can be seen clearly in figure 7. Figure 7 is a graphic representation of the cycle times of minimum idle time as determined by the digital computer iteration. The mechanical efficiency of the production line will decline in view of the fact that the idle time index value in the work stations increases from 16 000 to 22 000. This means that the excess 1,4 per cent of idleness is switched over to permanent idleness and that the safety factor disappears. As far as physical production is concerned, it is better if the production line functions at a higher mechanical efficiency with a built-in safety factor of 1,4 per cent on the demand period, than at a lower mechanical efficiency with no safety factor.

The solution to the second scheduling algorithm is that a production cycle of 44,35 and 18 per lot of the respective model types 1, 2 and 3 is alternated with a production cycle of 44,35 and 17. This will ensure that the average demand period of model type 3 is also approximately 790 minutes.

COMPARISON OF THE TWO SCHEDULING ALGORITHMS

The results obtained with the two scheduling algorithms are both positive. In both cases a schedule is developed which conforms to market needs.

In the schedule developed with the first algorithm, the inventory situation is such that stock of each model type is not available at all times. From a practical point of view this advantage is not serious because the out-of-stock situation losts far less than a day. The advantage of this method is that it is not necessary to specify a production range because the economic lot sizes which are manufactured, are precisely as determined. The reason for this is that the demand rate and production rate are tuned in to one another, and the inventory situation is controlled by an ever changing sequencing of the manufacturing lots of the various model types.

In the schedule developed with the second algorithm stock of all the model types is always available, but here the disadvantage lies in the deviation from the prescribed economic lot sizes which are manufactured. As in the case of the first scheduling algorithm, the demand rate and production rate are once again tuned in to one another because no change is made in the cycle time of the production line. The sequence in which the various lots of the model types are manufactured, remains the same.

Since the two methods yield the same result, namely that the needs of the market are satisfied without over or under provision, this makes the choice between the two scheduling algorithms a difficult one. This statement is made for two reasons. First, the out-of-stock situation of the first method cannot influence the optimality for the practical reasons already described. Secondly, the deviation of the prescribed economic lot sizes in the second scheduling algorithm only results in a slight increase in the variable cost of the model types. Both disadvantages connected with the two methods thus exercise no influence worth mentioning on the production process. Because the cycle time of the production line remains unchanged and the needs of the market are satisfied exactly by both scheduling methods, the total idle time originating throughout a schedule period is also the same for both scheduling methods.

When all the factors are taken into account, the first scheduling method is better in the respect that no increase in variable cost occurs. The out-of-stock situation or under provision of the first method has no influence on cost because it only lasts for a few hours at a time, and orders are carried out as has already been described. In most cases the orders of clients take place on a constant-with-order basis, which means that the units are already paid for, depending on whether or not they are in stock. The fact that a large part of the turnover takes place on this basis, means that the advantage of always having units in stock is to a large extent neutralized because no loss of lost order can be sustained. The first scheduling algorithm thus appears to be slightly better than the second.

THE MULTI-MODEL ALGORITHM — A FINAL SUMMARY

The complete algorithm as discussed in four publications in Business Management consists of the following steps:

- first, an optimum cycle time is found for the paced multimodel production line by minimizing idle time which originates throughout the total production schedule;
- secondly, optimum lot sizes are calculated for each model type which is manufactured on the paced production line;
- thirdly, two methods are proposed for the drawing up of an efficient production schedule; and
- fourthly, an optimal balancing of the task content of the work stations in the paced production line is performed for each model type which is manufactured.

The algorithm is unique in the sense that it first of all creates the possibility of determining an optimum cycle time in terms of idle time which originates throughout the total schedule, as a function of the respective market demands for the model types; and secondly, creates the possibility of determining the optimum cycle time and of performing production scheduling without making it necessary to carry out line balancing as well.

The line balancing technique is practical, relatively simple and generates a working balance of the task content per work station as based on the optimum cycle time. An important result which is discovered is that the duration of the line balancing programme on the digital computer declines sharply when an acceptability limit of only plus 0,5 per cent is allowed in the optimum cycle time. This step has no adverse influence on the efficiency of the line balance, and at the same time is advantageous in that planning costs are thereby saved.

By performing a sensitivity analysis we find that the algorithm is insensitive to a fluctuation in market demand in respect not only of minimum idle time-cycle times but also of optimum lot sizes of the model types which are manufactured. As a result of this, the practical value of the multi-model production technique is great in that no reorganization of the production plant or labour force is necessitated by a limit fluctuation in market demand. What is also indicated is that a minimum idle time-cycle time may occur in more than one cycle time value, and that this result can be applied beneficially where an increased or lowered production rate is required.

Of the two scheduling algorithms proposed, the first method has the attribute that although stock on hand of each model type is not available at all times, it is regarded as the best because there is no deviation from the specified optimum lot sizes of the model types. The out-of-stock situation can be accepted because it only lasts for a short time, and because orders are carried out in such a way that it is not necessary that there is always stock on hand of each model type.

Because the multi-model algorithm is programmed as a whole for application on the digital computer, its usefulness is increased. The advantage of digital computer application lies especially in its high accuracy and rapid performance. High accuracy and rapid performance have a particular bearing on line balancing and scheduling, because both are comprehensive methods which take up a great deal of time if performed manually.

Because competition in business life is on the increase and because methods whereby productivity can be increased are always sought after, more and more businesses which use the one-product-one-line method will be forced to turn to lot production on a paced production line. The algorithm provides such enterprises with the opportunity of performing an optimum change-over to lot production in situations where they were uncertain as to the precise method which should be used, and where they were uncertain of the possible consequences of such a step.

For business which do not employ lot production it is still beneficial since the line balancing technique can be applied separately from the rest of the algorithm, and because it presents an efficient method of balancing single model-production lines.

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