Comparing two computer search models for aggregate production planning

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In this paper a comparison is made between the results and the cost-effectiveness of two computer search models for aggregate production planning when applied to a very sensitive high-order cost structure. The Search Decision Rule (SDR) model developed by Taubert outperforms the Sectioning Search Model (SECT) of Goodman in both the areas of total optimum cost. and cost-effectiveness.

S. Afr. J. Bus. Mgmt. 1982, 13: 67 – 69

In hierdie artikel word 'n vergelyking getref tussen die resultate en die koste-doeltreffendheid van twee rekenaarsoekmodelle vir geheelskedule-produksiebeplanning, soos toegepas op 'n baie sensitiewe hoë-orde kostestruktuur. Die 'Search Decision Rule'-model (SDR) ontwikkel deur Taubert lewer in albei areas, naamlik totale optimale koste en kostedoeltreffendheid, beter resultate as die 'Sectioning Search Model' (SECT) van Goodman.

S.-Afr. Tydskr. Bedryfsl. 1982, 13: 67 – 69

Second in a series of three articles

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Received October 1981; accepted November 1981

Introduction

Many production managers are faced with the problem of planning production, inventory and work force under the constraint of limited recources to meet a seasonal demand. In those cases where linearity of the cost functions of an undertaking may reasonably be assumed, an ordinary linear programming model suffices. In many cases, however, this simple linear approach to certain essentially non-linear cost functions is unacceptable owing to the gross approximation made. Considerable research has been done on this planning problem and various models have been proposed. These models can be divided into three broad categories, namely heuristic models, mathematical optimization models and computer search models.

In this paper a comparison is made between the results of two of the published computer search models on a high-order cost function. One of the following four basic strategies can be followed to meet the fluctuations in demand.

1. Work-force level and production rate are kept constant and inventory is used to absorb fluctuations in demand.
2. Work-force level and inventory are kept constant and demand fluctuations are handled by changing the production rate, i.e. working overtime or allowing idle time.
3. Production rate and inventory are kept constant and the work force is varied to suit the demand.
4. A combination of the three strategies given above.

In most cases in industry the combination type of strategy (4) is usually the most appropriate. The extent to which the different strategies should be mixed to present an overall plan is dependent on the cost structure of the particular industry.

Cost structures vary, and may have anything from linear or almost linear, to highly non-linear relationships. In many cases ordinary linear or piecewise linear functions may be adequate to describe the relationship between cost and one of the above-mentioned variables. On the other hand it may well be that for certain costs a linear approach is unrealistic and far removed from the real world situation. In the latter case it becomes extremely difficult to obtain a proven optimum solution. Various methods have been suggested to solve this problem.

For a solution method to be practical it must comply with the following primary properties:

- It must be cost effective.
- It must assure, with reasonable confidence, that a global optimum will be reached.
- It must be universally applicable.

With the development of the high-speed digital computer, computer search methods have been developed and implemented to comply, in the field of aggregate production planning, with these properties. Taubert[1] compared various search algorithms and found the Hooke-Jeeves algorithm[2] particularly suitable for the solution of high-order functions. He made use of this algorithm in the development of his computer search system, Search Decision Rule (SDR). Goodman[3] applied a modified Sectioning Search Model (SECT) to a high-order cost function.

The author applied the SDR model to the high-order cost function used by Goodman and compared the results with those of the SECT.

Description of the cost structure

In order to test the Sectioning Search Model, Goodman developed a fourth-order cost model. The real world costs, of which this model is an approximation, are given in Table 1. The cost components considered in this model are: direct payroll, overtime and idle time, hiring and lay-off, change of production rate and inventory holding and shortages. The objective cost function to be minimized is:

$$C = \sum_{t=1}^{N} [340\,W_t + 0{,}2\,(P_t - 6W_t)^4 + 64(W_t - W_{t-1})^4 + 0{,}1\,(P_t - P_{t-1})^4 + 0{,}1(320 - I_t)^4]$$

Where W_t is the work force in period t,
P_t is the production in period t, and
I_t is the inventory in period t;
subject to the following constraints:

$$I_t = I_{t-1} + P_t - D_t \qquad (t = 1 \text{ to } N)$$

$$0 \leqslant W_t \leqslant 150 \qquad (t = 1 \text{ to } N)$$

$$0 \leqslant P_t \leqslant 1\,000 \qquad (t = 1 \text{ to } N)$$

where D_t is the demand in period t.

Work force and production quantity in each period are the independent variables. From these variables, as well as the given demand (D_t), the other variable contributing to the cost, that is inventory, is calculated.

Results

In Table 2 the monthly production plans and corresponding costs given by SDR and SECT are compared for a 24-month planning horizon. SDR gave an improvement of nearly 6% on the total cost of $14 196 488 obtained by Goodman's SECT. It can also be seen that the cost model is very sensitive to small changes in any of the variables — compare for example the monthly costs for months 3 (SECT 53% higher than SDR), 4 (SECT 52% higher than SDR) and 8 (SECT 52% higher than SDR). There are no major differences between the two plans. From a practical point of view any one of the two plans could be adopted. It must thus be emphasized that for highly-sensitive cost structures as the one used here, extreme care must be taken in the choice of an optimization method.

To measure the cost-efficiency of the two search techniques, the computer time required per decision (independent variable) is compared. It should be kept in mind, however, that as Goodman states, '. . . computer time usage is a function of both the computer used and programming efficiency and method used'.[3] He states that on average the Sectioning Search Model uses 0,75 s per decision. It was found that SDR used only 0,38 s per decision on a UNIVAC 1110 computer. By decreasing the number of search repetitions of the SDR procedure, a plan was obtained using only 0,24 s per decision (a 68% saving in computer time). The total cost of this plan was only 0,14% higher than the results previously obtained.

Conclusion

In this paper a comparison is made between the results of two well-known search models developed for aggregate production planning. For comparison purposes a high-order cost model has been used. The SDR-model of Taubert outperforms the SECT-model of Goodman when compared on the basis of total optimum cost and cost-effectiveness.

Table 1 Real world cost on which the cost model is based (Rand)

$\lvert W_t - W_{t-1}\rvert$	Cost	$\lvert P_t - P_{t-1}\rvert$	Cost	$\lvert I_t - 320\rvert$	Cost	$\lvert P_t - 6W_t\rvert$	Cost
0	0	1	1	1	1	0	0
1	66	2	2	2	2	1	1
2	1001	4	24	3	9	2	4
3	5210	5	68	4	28	3	14
4	20100	7	225	6	122	5	131
5	38120	10	1049	8	392	7	457
7	86300	16	6310	11	1370	10	1876
9	139200	22	26100	15	5417	12	3780
12	224400	34	123400	21	18240	14	7795
14	279600	52	487200	39	231200	18	20600
19	401100	87	1140000	51	474400	22	34900
25	698700	150	2224000	70	762500	30	58200

Table 2 Production plans, Search Decision Rule (SDR) and Sectioning Search (SECT)

Period	Demand	Work force SDR	Work force SECT	Production SDR	Production SECT	Inventory SDR	Inventory SECT	Period cost SDR (Rand)	Period cost SECT (Rand)
1	430	73	75	447	431	337	301	36205	160572
2	447	70	72	431	440	321	294	36187	76836
3	440	67	69	406	426	287	280	182702	392632
4	316	64	65	376	392	347	356	163305	340082
5	397	62	62	362	374	312	333	29178	39620
6	375	60	59	352	348	289	306	109783	75042
7	292	62	60	364	348	361	362	305626	335780
8	458	64	63	395	386	298	290	151989	316936
9	400	63	64	383	391	281	281	244834	253710
10	350	61	61	353	355	284	286	273730	330447
11	284	63	63	359	356	359	358	278352	277808
12	400	68	68	399	399	358	357	532877	593728
13	483	73	73	442	444	317	318	414825	475143
14	509	78	78	477	481	285	290	362126	340645
15	500	83	83	491	488	276	278	432210	381629
16	475	88	88	510	508	311	311	103618	118576
17	500	94	94	553	552	364	363	853346	835740
18	600	101	101	608	607	372	370	1752367	1728065
19	700	107	107	662	662	334	332	1090724	1068460
20	700	112	112	698	699	332	331	315675	373248
21	725	107	107	658	659	265	265	1273906	1264146
22	600	101	101	600	600	265	265	2171033	2244341
23	432	95	95	545	545	378	378	2222244	2240081
24	615	93	93	552	553	315	316	32637	33204
Total cost								13368479	14196458

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