A probabilistic approach to the evaluation of risk-related investments with reference to their location in industrial development/deconcentration points in South Africa

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The policy and programme for *industrial decentralization* in South Africa forms an integral part of South Africa's total economic development strategy for the future. Therefore, in the wake of South Africa's revised regional economic development proposals — which have resulted in the introduction of predominantly *cash-based* industrial decentralization incentives relative to their predominantly tax-based precursors — the author purports to outline the tenets underlying a probabilistic approach to the evaluation of risk-related investments with reference to their location in industrial development/deconcentration points in South Africa. To this end, the author seeks to illustrate that in evaluating capital investment proposals — within the context of regional decentralization — *cash flow streams* are one of the principal determinants of project worth in the analytical process. Moreover, although much of contemporary capital budgeting work is based on assumed 'conditions of certainty' a probabilistic approach to cash flow formulations is adopted in this article in the conviction that this affords considerably more insight into the problems of project evaluation and optimal selection.
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Die nywerheidsdesentralisasiebeleid en -program in Suid-Afrika is 'n integrale deel van Suid-Afrika se algehele ekonomiese-ontwikkelingstrategie vir die toekoms. As gevolg van Suid-Afrika se hersiene voorstelle vir ekonomiese ontwikkeling in die onderskeie streke, het hoofsaaklik kontantgebaseerde aansporings tot nywerheidsdesentralisasie hulle belastinggebaseerde voorlopers (grootliks) vervang. Na aanleiding hiervan wil die skrywer die onderliggende beginsels van 'n probabilistiese benadering tot die evaluering van risikoverwante beleggings met betrekking tot hulle vestiging in nywerheidsontwikkelings-/-dekonsentrasie gebiede, in breë trekke bespreek. Vir dié doel probeer die skrywer verduidelik dat, by 'n evaluering van kapitaalbeleggingsvoorstelle teen die agtergrond van streekdesentralisasie, *kontantbewegingstrome* een van die vernaamste beslissende faktore is ten opsigte van projekwaarde in die analitiese proses. Aangesien die opstel van 'n kapitaalbegroting deesdae grootliks op veronderstelde 'sekerheidsvoorwaardes' gegrond is, word daar verder in hierdie artikel 'n probabilistiese benadering tot kontantbewegings-formulerings ingeneem, met die oortuiging dat dit groter insig bied in die problematiek van projekevaluering en optimale keuring.
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Regional development incentives: A cash-based perspective

With regard to general principles, the incentives of the new policy are interesting and potentially important for three reasons. The first follows from the differences in the priority rating of the regions concerned, inasmuch as the level of incentives increases in accordance with an increase in priority level. Secondly, the existence of long-term (permanent) cost disadvantages of locating in decentralized areas is explicitly acknowledged through the general distinction which is made between incentives aimed at compensating the industrialist for permanent disadvantages and incentives aimed at alleviating certain short-term financing problems. In effect, this acknowledgement is tantamount to an acceptance by the authorities of the fact that the permanent cost disadvantages associated with locating at specific development points, together with the corresponding long-term incentives, represent a permanent tax on the community. However, as McCarthy (1982:248) argued, it amounts to an *ex-ante* admission of the existence of inappropriate cost-ineffective locations.

Thirdly, incentives vary within demarcated regions at different development points and consistently favour the development points located in the National States.

This differentiation is justified by the possibility that certain development points may qualify for higher incentives 'to meet specific needs and priorities, especially points in the independent and self-governing National States which, for various reasons, have particular locational disadvantages' (Good Hope Plan, 1981:76). Indeed, this variation in incentive levels is based on the belief that 'industrial development inside the independent and self-governing National States should be the first priority' (McCarthy, 1982:249). Industrial development areas have, therefore, been chosen in such a way that industrial development outside the independent and self-governing States would not be promoted at the expense of development inside those States (Good Hope Plan, 1981:72 – 73).

According to the foregoing distinction, the new incentives may be classified as *labour incentives*, which include the employment incentive (the wage subsidy) and the training grant, both of which take the form of non-taxable *cash grants*, and the housing subsidy. *Capital incentives*, in turn, comprise rental and interest subsidies for industrial investments and an interest subsidy on infrastructural investment. *Neutral incentives*, on the other hand, include transport rebates, an electricity subsidy, and the re-location allowance (Du Toit, 1982:255).

It may, therefore, be inferred that the regional economic development proposals are biased in favour of labour intensive industries with limited capital requirements. To this end, the new scheme of incentives favours the creation of jobs, as reflected in the composition of the new incentives.

Consequently, it is with this in mind that those incentives which stimulated the use of capital relative to labour, such as the 30% income tax concession, based on the investment in plant and machinery as well as the enhanced initial and investment allowance on buildings, plant and machinery, have been abolished.

Moreover, it may be concluded that although the new incentives have not undergone any definitive changes exclusively in favour of job creation, the *cash grants* coupled to job creation nevertheless emphasize this objective. What is more, there has been a decisive swing towards more neutral incentives which are applicable to all industries over the long-term.

A further noteworthy feature of the new dispensation relates to the small industrialist (in terms of capital-labour ratios) who stands to gain from the general emphasis on improving the short-term financing position of beneficiaries under the third schedule of concessions, which became operative on 1 April, 1982. This is to be achieved by the use of *cash grants* instead of tax rebates. Herein lies the emphasis which is germane to this article, namely, the advocacy of a probabilistic approach to the evaluation of capital investment proposals. In essence, this involves an exact *means-variance* analysis of the distinctive cash flows associated with a capital investment proposal, the execution of which is subject to the constraints of South Africa's regional economic development programme.

Indeed, this approach is facilitated by the fact that all the short-term incentives are being made available as *cash payments* and are aimed at helping to bridge the *cash flow* problems of industrialists, particularly during the establishment phase of their factories. By contrast, it should be noted that previously certain short-term measures were in the first place granted in the form of tax concessions which were only converted to cash allowances in the event of an industrialist finding himself in a loss situation. Therefore, the third schedule of concessions clearly represents an improvement in this regard.

The exact means-variance approach

The statistical technique proposed to be used for purpose of evaluating capital expenditure proposals is the exact means-variance analysis of the probability distribution of the corresponding net present values and internal rates of return — as optimal profitability criterion functions — derived from an autocorrelated linear stream of random net cash flows. These cash flows per time period are indicative of the anticipated profitability and, therefore, the degree of riskiness likely to accrue from the proposed implementation of a capital expenditure programme, given the alternative locational constraints associated with a decentralized industrial development point.

The statistical definition of these two criterion functions may be stated as follows (Wagle, 1967:3):

- The net present value of a proposed investment is defined as the difference between the discounted future earnings and the current investment. The discounting factor is often described as the cost of capital.
- The internal rate of return is that rate of interest which, when applied to the various cash flows over the life of the investment, treating outflows as negative and inflows as positive, gives a zero present value.

The foregoing in effect represents a *probabilistic approach* which seeks to take into account the uncertainty surrounding the variables which enter into a locational feasibility study by considering their probability distributions. However, it invariably aims to provide the probability distribution of the dominant profitability criterion function, relative to a capital expenditure proposal, notably, NPV. To this end, the approach summarizes into a single figure the economic desirability of a proposed project.

Normally, this estimate is calculated on the basis of the most likely values of the individual factors that make up the respective profitability criterion functions. However, as these factors are themselves subject to uncertainty this is carried on into the profitability criterion function. For instance, an IRR estimate may be compiled on the assumption of a 3% market growth rate, a 2% rise in prices and a 5% fall in costs. On the basis of these estimates the IRR may come out very high. However, this assumes that the most likely estimates will materialize, that is, that there is no uncertainty in them. This optimism is practically unrealistic inasmuch as any of the variables that have a bearing on IRR could quite possibly take on lower values, leading to a much lower value of IRR.

Consequently, in considering a capital investment proposal an assessment of the risk involved is necessary. The term risk refers to the potential for a project's return to fail to achieve any given rate, usually determined by a company's prevailing hurdle rate. This is because the amount of risk involved must be treated as one of the fundamental considerations in the evaluation of proposed investments. Therefore a reasonably safe investment with a certain expected rate of return will often be preferred to a much more risky investment with a somewhat higher expected rate of return. This is especially true when the risky investment is so large that the failure to achieve expectations could significantly affect the financial position of the individual or firm (Hillier, 1963:443).

Allied to the assessment of this risk is the need for explicit, well-defined and comprehensive information with a view to effecting an accurate appraisal of the potential risk in an investment. Inasmuch as such information is frequently not available in the required format, it is proposed to illustrate how such information, in the form of the probability distributions of IRR and NPV, can be derived.

In practice, this information is usually *approximated* with the aid of techniques such as sensitivity analysis and the Monte Carlo simulation method. However, with this article the author proposes to supersede both these techniques.

In so far as sensitivity analysis is concerned, its primary aim is to examine the effects on the profitability criterion function of changes in the values of the key economic variables. A particular case of sensitivity analysis is to take high, low and medium values of key economic parameters and compute the value of the criterion function for various combinations of these pessimistic, average and optimistic estimates, thus providing a range of possible results. Although this method gives some useful information it suffers from the weakness that it does not provide any measure of the likelihood of obtaining any particular value of the criterion function (Wagle, 1967:14).

In short, sensitivity analysis is quite limited in the amount of information it can provide. Hence, it is difficult to draw precise conclusions about the possible effects of combinations of errors in the estimates, even though this is the typical situation of concern to management faced with a capital investment proposal. Moreover, for statistical reasons, it would usually be misleading to consider the case where all the estimates are too optimistic or where all are too pessimistic. Therefore, although sensitivity analysis is useful, its conclusions tend to suffer from a lack of conciseness, precision and comprehensiveness (Hillier, 1963:444).

Likewise, the Monte Carlo simulation method exhibits similar practical limitations. This is particularly true in the case of proposed approach, which is predicated on an exact means-variance analysis of various cash flows emanating from different sources, notably, the sales cash flow, the variable cost cash flow and the fixed cost cash flow, all of which are peculiar to a locational feasibility study. However, in many situations these cash flows may not be known directly. Rather, what is usually available are the means and variances of the factors which make up each of these cash flows.

Therefore, because each cash flow in any year is a function of several variables each of which has its own probability distribution, it may happen that the calculation of the mean (μ) and the standard deviation (σ) of the profitability criterion function may be different. It is precisely at this juncture that the Monte Carlo method has its appeal. This is because of its ability to derive the approximate probability distribution of the profitability criterion functions (Wagle, 1967:15).

However, this method should be viewed with circumspection, despite the advocacy for its use by proponents such as Hess & Quigley (1963:55 – 63) and Schreider (1967:10 – 17). The reason for this caveat is because the Monte Carlo method only provides an approximation of both the probability distribution and the corresponding parameters of the profitability criterion function such as NPV and IRR. Moreover, the method necessitates considerable programming work (Wagle, 1967:31).

In essence, because the Monte Carlo simulation method is a cumbersome, iterative process, it is frequently perceived by statisticians to be inferior to the rigours of risk analysis based on probabilistic information, as presented in this article, with a view to enabling the locational decision-maker to understand and evaluate uncertainty.

The probability distribution of NPV and IRR

In view of the uncertainty surrounding the variables entering into a locational feasibility analysis relating to a capital investment proposition, it is proposed to generate and evaluate the probability distributions of the appropriate profitability criterion functions, notably, NPV and IRR. These are frequently used to assess the merits of such a proposition, given the alternative constraints which have a bearing on such a proposal, notably, those of a locational nature.

Inasmuch as the cash flows of an investment commonly emanate from a number of distinct sources (for example, an investment may affect sales income, labour costs and the like), it would facilitate determining the pattern of variations of the resultant net cash flows and their corresponding correlations if these distinct sources were treated separately.

Notationally, this may be effected as follows. Let there be m sources of cash flows emanating from an investment. Let the *random variable* $Y_{i\alpha}$ denote the cash flow in period i from the αth source. Moreover, assume that $Y_{i\alpha}$ has a finite mean, $\mu_{i\alpha}$, and variance, $\sigma^2_{i\alpha}$. Then by allowing the net cash flow in the ith period to be denoted by X_i, the following formulae obtain, as postulated by Wagle (1967:16).

$$X_i = Y_{i1} + Y_{i2} = \ . \ . \ . \ Y_{im} \qquad (1)$$

Consequently, for the probabilistic case each periodically generated net cash flow increment is conceived of as a *random* variable, either discretely or continuously distributed over the range of interest or applicability, rather than as a 'known' constant value for any given period. The consequence of this distributional assumption is that each random cash flow increment, $Y_{i\alpha}$, and each time period i, will have at least a *mean* and a *variance* associated with it, and possibly higher central moments as well although for the purpose of this article the author will confine himself to the case in which only the means and variances of the cash flow increments are of interest.

Therefore, taking expectations (means) and variances, the following holds:

$$E(X_i) = E\,(Y_{i1} + Y_{i2} + \ . \ . \ . \ Y_{im}) \ . \ . \ . \text{ from equation} \qquad (1)$$

$$= (\sum_{\alpha=1}^{m} Y_{i\alpha}) = \sum_{\alpha=1}^{m} E(Y_{i\alpha}) \ . \ . \ . \text{ the expected value}$$

of the sum equals the sum of the expected values

$$= \sum_{\alpha=1}^{m} \mu_{i\alpha}$$

$$\text{var}(X_i) = \sum_{\alpha=1}^{m} \sigma^2_{i\alpha} + 2\sum_{\alpha\neq\beta} \text{cov}(Y_{i\alpha}, Y_{i\beta}) \qquad (3)$$

where $\sigma^2_{i\alpha}$ = the variance of $Y_{i\alpha}$ and $\text{cov}(Y_{i\alpha}, Y_{i\beta})$ = covariance of $Y_{i\alpha}$ with $Y_{i\beta}$.

If the cash flows last over n periods then *the present value of this investment* is defined as follows:

$$\text{NPV}_n = d^0X_0 + d^1X_1 + d^2X_2 + \ . \ . \ . \ d^nX_n$$

$$= \sum_{i=0}^{n} d^iX^i \qquad (4)$$

where $d^i \equiv$ discounting factor for NCF (net cash flow) in financial period $i = (\frac{1}{1+k})^i$ and $X_i \equiv$ net cash flow in period i; $k \equiv$ discount rate.

In presenting the expression for the mean (expected value) and variance of NPV_n below, the author proposes to deal, firstly, with the case where each discounting factor (d^i, i = 0, 1, 2, . . ., n) is considered to be a constant. Consequently, the following holds:

$$E(\text{NPV}_n) = E[\sum_{i=0}^{n} d^iX_i] \ . \ . \ . \ d^i \text{ is a constant.}$$

$$= \sum_{i=0}^{n} d_i\, E(X_i)$$

$$= \sum_{i=0}^{n} \frac{E(X_i)}{(1+k)^1}$$

and

$$\text{var}(\text{NPV}_n) = \sum_{i=0}^{n} \frac{\text{var}(X_i)}{(1+k)^{2i}} + 2\sum_{i\neq i^1} \frac{\text{cov}(X_i, X_i^1)}{(1+k)^{i+i^1}}$$

However, the author also proposes to treat each discounting factor (d^i; i = 0, 1, 2, . . ., n) as a normally distributed random variable. Therefore it follows that each term d^iX_i in equation (4) is the product of two random variables. Moreover, it is assumed that both these random variables are normally and mutually independently distributed. Indeed, each term in equation (4) is now considered a composite of two constituent variables. In order to find the mean and variance of NPV_n now, it is necessary to find the mean and variance of each term in equation (4).

To this end:

$$E(d^iX_i) = E(d^i) \ . \ E(X_i) \text{ and} \qquad (7)$$

$$\text{var}(d^iX_i) = [E(d^i)]^2 \ . \ \text{var}(X_i) + [E(X_i)]^2 \ . \ \text{var}(d^i) + \text{var}(d^i).\text{var}(X_i) \qquad (8)$$

It is however evident that $E(d^i)$ and $\text{var}(d^i)$ in equations (7) and (8) above are not immediately known. Rather, they are determined according to the same logic; namely the derivation of the mean and variance of a composite variable that is the product of two mutually independent and normally distributed random variables, as illustrated below.

$E(d^2) = E(d).E(d)$ .... from equation (7)

or

$E(d^3) = E(d^2).E(d)$

$\text{var}(d^2) = 2.[E(d)]^2.\text{var}(d) + [\text{var}(d)]^2$ from equation (8)

or

$\text{var}(d^3) = [E(d^2)]^2.\text{var}(d) + [E(d)]^2.\text{var}(d^2) + \text{var}(d)\text{var}(d^2)$

etc.

Therefore, the random net present value for a project possesses a *mean net present value* $E(\text{NPV}_n)$ and a *variance of net present value*, $\text{var}(\text{NPV}_n)$. These are the keys that relate the unknown NPV_n to the random cash flow increments of a project (Bussey & Stevens, 1972:10).

Moreover, if it is assumed that the distribution of net present value is normal, then using the identity that:

$$\text{prob}\{r < k\} = \text{prob}\{P_n(k) < 0/k\} \quad (9)$$

where the internal rate of return, r, is defined as that value of k for which $\text{NPV}_n = 0$, it is possible to derive the *cumulative distribution function* of r. From this cumulative distribution function, the probability density function can be readily obtained although this may not always be necessary.

Some of the conditions under which NPV_n will be normally distributed are the following, as indicated by Hillier (1969: 25 – 29).

- If $X_0, X_1, \ldots X_n$ have a multivariate normal distribution, then NPV_n, being a linear function of the X's, would itself be normally distributed.
- Because NPV_n is the sum of a number of random variables, it follows by the central limit theorem that under certain conditions NPV_n is asymptotically normally distributed. The best known version of the central limit theorem states that if a set of random variables, $W_1, W_2, \ldots W_n$, are independent and identically distributed with finite mean and variance, their sum is asymptotically normal.

However, an essential theoretical difficulty is that NPV_n is not the direct sum of random variables, but rather the weighted sum, in which the weights are the discounting factors. The effect of this is that the shape of the distribution of NPV_n may be dominated by early cash flows, especially at a high discount rate. Therefore, in the case of independently distributed cash flows continuing for ever, the variance of the present value of the first n cash flows would remain finite as $n \rightarrow \infty$, and in this case it is known that the distribution of the net present value will not tend to normality unless each of the net cash flows is normally distributed. However, because the net cash flows may themselves be (explicitly or implicitly) sums of a number of variates, it may be reasonable to assume that they are normally distributed, thus circumventing this difficulty (Wagle, 1967:18). This assumption applies to the present article.

Estimation of means, variances and covariances of cash flows

In order to determine the probability distribution of NPV_n it is necessary to know the probability distribution (or at least the mean and variance) of each of the individual cash flows and the covariances between them. Indeed, the question of how to determine the probability distribution of each of the cash flow increments, $Y_{i\alpha}$, for a project is a question of major importance because the distribution of each periodic cash flow increment forms the basic data inputs to the entire capital budgeting problem.

In a practical setting this may require that an analyst describes a subjective probability distribution for each of these cash flows, as illustrated in Appendix A. To this end it becomes necessary to effect three readily comprehendable types of estimates which will completely determine the specific probability distribution for each cash flow, reflective of the source elements that contribute to cash outflows and inflows.

According to Hillier (1969:87 – 89), the suggested estimating procedure is to apply an 'optimistic' estimate, a 'pessimistic' estimate, and a 'most likely' estimate to each of the source elements. To this end it is possible to develop a mean (expected value) estimate for the net cash flow in a given period. Furthermore, by virtue of the probabilistic approach to this exercise, it is also necessary to somehow realistically (albeit subjectively) evaluate the variance of the net cash flow increment for each period of interest, or alternatively, specify the probability or density function of each net cash flow increment, as reflected in Appendix A.

Consequently, the obvious method of estimating these parameters would be through a series of meetings with management to describe the probability limits for these estimates. It is, moreover, assumed that these estimates correspond to the lower bound, upper bound and mode, respectively, of the probability distribution. It is further assumed that an adequate model for the form of this distribution is the beta distribution such that the standard deviation is $^1/_6$ of the spread between the lower bound and upper bound. This assumption is tenable if it is accepted that the beta distribution somewhat resembles the normal distribution.

Therefore, under the assumption that each cash flow has a beta distribution with a spread of six standard deviations between the bounds, the mean and variance are explicit functions of the bounds and the mode. Accordingly, the mean and variance of the cash flow increment in any period i, which typifies the beta distribution, can be found by using the following generic expression:

$$E[Y_{i\alpha}] = {^1/_6}\,[\text{Est}(Y_p) + 4\ \text{Est}(Y) + \text{Est}(Y_0)]$$

$$\text{var}\,[Y_{i\alpha}] = \{{^1/_6}\,[\text{Est}(Y_0) - \text{Est}(Y_p)]\}^2$$

where $E[Y_{i\alpha}]$ = mean cash flow increment for period i;
$\text{var}[Y_{i\alpha}]$ = variance of the cash flow increment for period i;
$\text{Est}(Y)$ = 'most likely' estimate of cash flow in period i;
$\text{Est}(Y_p)$ = 'pessimistic' estimate of cash flow in period i;
$\text{Est}(Y_0)$ = 'optimistic' estimate of cash flow in period i.

Hence, if these estimates are denoted by m, b and a respectively, the mean of the corresponding distribution is $(a + b + 4m)/6$ and the standard deviation is $(a - b)/6$ as stipulated by Wagle (1969:19).

What should be noted, however, is that although an underlying beta distribution may be assumed for each cash flow, the actual underlying distribution may be essentially unbounded (Wagle, 1967:20), which could lead to an extremely large estimate of the standard deviation (σ). In fact the location is not important, rather the extremely vital requirement is that the spread between the pessimistic and optimistic estimates should represent six standard deviations for the actual distribution.

To illustrate this technique reference will be made to the estimates relating to sales and price and the correlation between them, with a view to determining the mean and variance of the corresponding sales cash flow.

Let U_1 denote sales with mean η_1 and variance σ^2, and U_2 denote price with mean η_2 and variance $\sigma_2{}^2$. Let the correlation coefficient between U_1 and U_2 be denoted by ρ.

The foregoing presupposition permits the following cases to be considered:

(i) where U_1 and U_2 are independent random variables ($\rho = 0$):

$$E(U_1\ U_2) = \eta_1\ \eta_2 \quad (10)$$

$$\text{var}\ (U_1U_2) = \eta_1^2\sigma_2^2 + \eta_2^2\sigma_1^2 + \sigma_1^2\sigma_2^2 \quad (11)$$

and,

(ii) where U_1 and U_2 have a joint bivariate normal distribution with the result that $\rho \neq 0$:

$$E(U_1U_2) = \eta_1\eta_2 + \rho\sigma_1\sigma_2 \quad (12)$$

$$\text{var}(U_1U_2) = \eta_1^2\sigma_2^2 + \eta_2^2\sigma_1^2 + 2\rho\eta_1\eta_2\sigma_1\sigma_2 + \sigma_1^2\sigma_2^2\ (1 + \rho^2) \quad (13)$$

However, for the application of the foregoing formulations, which are based on equations (5) and (6), it is necessary to have estimates of all the correlation coefficients (covariances) between the various cash flows. But, as Hillier (1969:89) observed, this estimating procedure may be a prohibitively large task. Consequently he proposed the following patterns of correlations which are both reasonable and sufficiently simple to be compatible with the limitations of the estimating and computing procedures. This model of correlations will permit inferring the values for all of the correlation coefficients on the basis of estimated values for only a small proportion of them.

Therefore, let $Y_{i\alpha}$ (indicative of a random variable) denote the cash flow in the ith period emanating from the αth source. Then the following assumptions are made:

- Cash flows of the same type are Markov-dependent, that is, a cash flow in time period $(i-1)$ will influence a cash flow of *that kind* in period J where $J > i$ only inasmuch as this influence is carried over from time period i. In statistical terms the partial correlation coefficient between $Y_{i\alpha}$ and $Y_{i'\alpha}$ with respect to $Y_{i-1\alpha}$ is zero.
- The correlation of cash flows of the same kind in adjacent periods is constant over time. Under these two assumptions the following obtains:

$$\text{cor}\ [Y_{i\alpha},\ Y_{i'\alpha}] = \rho_\alpha[i' - i] \quad (14)$$

where ρ_α denotes the correlation between successive periods emanating from the αth source.

- The second type of correlation pattern which needs specification is the correlation between cash flows of different types. Here the assumption is that the cash flow in a stated time period given a different type of cash flow in the same period is independent of the latter type of cash flow in an earlier period. This is quite realistic because circumstances which tend to push several types of cash flows up or down would tend to affect these cash flows simultaneously rather than in different time periods.
- That the correlation coefficient between cash flows emanating from two sources in a given time period is independent of the time period, for example, $Y_{i\alpha}$ and $Y_{i\beta}$ have a constant correlation $\mathring{\rho}_{\alpha\beta}$ over all i.

On the basis of this assumption it can easily be shown that the correlation coefficient between $Y_{i\alpha}$ and $Y_{i'\beta}(i > i')$ is given by:

$$(\rho_{\alpha\beta})\ (\rho_\beta \mid i' - i \mid) \quad (15)$$

Therefore, in order to use the above results the following basic correlation coefficients are required:

- Correlation coefficients between cash flows in successive time periods from each source, namely, $\rho\alpha$'s.
- Correlation coefficients between cash flows in the same time period, for example $\mathring{\rho}_{\alpha\beta}$.

Calculations of the expectations and standard deviations of the various cash flows

Introduction

As already mentioned, the variables peculiar to this approach are summarized in Appendix A together with their most likely values and corresponding ranges. This summary reflects an actual empirical application of this approach. From this it is possible to calculate the expectations (μ) and standard deviations (σ) of the various cash flows for any number of chosen time periods (financial quarters). In so far as the revised decentralization incentives are concerned the most obvious time periods would be within the range of 1 – 28 financial quarters, after which the non-taxable cash-based labour incentive subsidy expires, or between the 29th and 40th financial quarters, after which the taxable cash-based interest and rental subsidies expire or any period thereafter when all the cash-based decentralization incentives have expired, as stipulated in the third schedule of concessions.

Moreover, for the sake of expediency these demonstrative calculations assume that the cash flows in different time periods are independent of each other.

It may, however, be more pedantic to assume that the cash flows are constantly correlated across time periods, which assumption applies to the proposed approach.

Estimation of sales cash flow

Given that the total size of the initial market and its growth rate, designated S_1, and S_3 in Appendix A, are both random variables (which are assumed to be independent) the use of equations (10) and (11) yields the mean (η) and standard deviation (σ) of the market size in any time period as inferred from the range of values stipulated in Appendix A.

Furthermore, as company sales is a product of total market size and market share, equations (10) and (11) can again be used to estimate the corresponding mean (η) and standard deviation (σ) of company sales.

In Table 1 the format according to which these calculations may be effected is revealed. The implication is that the individual sales cash flow items represent individual beta distributions but that collectively the summation of their randomness allows the composite sales cash flow variable, to approximate a normal distribution.

Finally, the cash flow emanating from company sales is the product of sales and price. On the assumption that these two factors are correlated, it is possible to obtain a final estimate of this correlation coefficient (ρ). Thereafter the use of equations (12) and (13) yields the mean (η) and variance (σ^2) of the sales cash flows in the chosen time periods (n). A partial summary of the results of the application of these equations appears in Table 2.

Estimation of cost cash flow

The next step is to calculate the means (η) and standard deviations (σ) of other cash flows, namely, variable costs and fixed costs. On the assumption that variable costs and sales are correlated it is possible to arrive at a final correlation coefficient (ρ) between variable costs and sales.

Once again the use of equations (12) and (13) will yield the mean (η) and standard deviation (σ) of the cash flows resulting from variable costs in the respective chosen time periods. However, the cash flow fixed costs is assumed to have a constant expected value (μ) and standard deviation (σ) over all time periods (n). Table 3 constitutes a partial summary of the results of the application of this calculation.

Table 1 Basic input variables for means-variance analysis of sales cash flow

Variables	Most likely Estimate (m)	Range ($b-a$)	Mean (μ) $(\frac{a+b+4m}{6})$	Standard deviation (σ) $[(a-b)/6]$
Total market size — captive market:				
Product A	57 000	54 500 – 58 000	56 750	583,3
Product B	4 087	4 008 – 4 169	4 087,5	26,83
Total market size: All end users				
Product A	500 000	480 000 – 575 000	509 170	15830
Product B	35 000	33 600 – 40 250	35 642	1108,33
Selling price | cents | Units				
Product A	22,92	20,37 – 25,79	22,97	0,9
Product B	17,65	15,6 – 19,86	17,68	0,71
Market growth rate — % P.A.	4	2 – 7	2 – 6	0,67
Sales to captive market				
Product A	31 246	29 932 – 32 618	31 255,67	447,67
Product B	2 352	2 255 – 2 455	2 353	33,33
Sales to outside (in the future)				
Product A	–	–	–	–
Product B	–	–	–	–
Price % changes: Average for product A and B	15	12,5 – 20	15,42	1,25

Table 2 Summary of means-variance analysis of sales cash flow[a] for year 1

	Product A		Product B	
	Mean	*SD*	Mean	*SD*
Total market size — captive market	56 750	58,33	408,75	26,83
Total market size — all end users	509 170	15 839	35 642	1108,33
Sales to captive market	31 255,67	447,67	447,67	2353
Sales outside	33,33	–	–	–
Sales volume	31 255,67	447,67	2353	33,33
Selling price	22,97	0,9	17,68	0,71
Sales cash	717 983,03	32 908,42	41 603,41	1 767,61

[a]Total sales cash flow: mean — 759586 and *SD* — 34676,03

Table 3 Partial summary of means-variance analysis of cost cash flow

	Variable costs						Fixed costs					
	Product A		Product B		Total variable cost cash flow		Product A		Product B		Total fixed cost cash flow	
Year	Mean	*SD*	Mean	*SD*	Mean	*SD*	Mean	*SD*	Mean	*SD*	Mean	*SD*
1982	436366,4	23661,15	2494,4	147,25	438860,8	23808,4	79701,96	6673,39	447,1	88,29	80149,06	6761,68
1983												
1984												
etc.												

Calculation of the parameters of the distribution of net cash flow in various time periods (n)

The use of equations (2) and (3) yields the mean (η) and the variance (σ^2) of the net cash flow for each chosen time period (n). However, as sales cash flow and variable costs cash flow are correlated with each other, some idea of this correlation coefficient is necessary before equation (3) can be used. This correlation estimate — across cash flows emanating from different sources — can normally be effected intuitively with the aid of historical empirical data and subject to the correlation patterns enunciated earlier on in this article.

The means (η) and standard deviations (σ) of a hypothetical set of cash flows in various time periods are summarized in Table 4 to illustrate the results of this calculation.

Calculation of the means and variances of the net present value NPV$_n$ for different rates and life of investment (n)

For different discount rates, and using equations (5) and (6), it is possible to estimate the mean (η) and variance (σ^2) of the present value NPV$_n$ conditional upon the cash flows lasting for a given number of time periods (n). A hypothetical set

Table 4 Means and standard deviations of various cash flows

	Investment costs		Sales cash flow		Variable costs cash flow		Fixed costs cash flow		Residual value		Net cash flow	
Year	Mean (000's)	*SD* (000's)	Mean (000's)	*SD* (000's)	Mean (000's)	*SD* (000's)	Mean (000's)	*SD* (000's)	Mean (000's)	*SD* (000's)	Mean (000's)	*SD* (000's)
0	9250	580									-9250	1000
1			13,412	3000	11,940	2910	304	20,8			1168	1871
2			13,814	3085	12,298	2996	304	20,8			1213	1930
3			14,228	3190	12,666	3092	304	20,8			1258	1900
4			14,652	3290	13,044	3191	304	20,8			1304	2052
5			15,093	3390	13,437	3285	304	20,8			1352	2110
6			15,546	3500	13,840	3388	304	20,8			1402	2179
7			16,009	3600	14,253	3492	304	20,8			1452	2245
8			16,490	3715	14,681	3603	304	20,8			1505	2317
9			16,987	3830	15,123	3712	304	20,8			1560	2390
10			17,495	3940	15,576	3823	304	20,8			1616	2458
11			18,021	4065	16,043	3942	304	20,8			1674	2535
12			18,562	4190	16,526	4064	304	20,8			1732	2613
13			19,121	4333	17,023	4194	304	20,8			1794	2700
14			19,696	4475	17,534	4326	304	20,8			1858	2783
15			20,288	4610	18,061	4468	304	20,8			1923	2874
Residual									4400	250	4400	250

Source: Wagle (1967:28)

Table 5 Means and variances of present value for different discount rates and life of investment

Life of investment		Discount rate (% p.a.) 0		2,5		5		10		12,5		15	
Years	Prob.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.	Mean	Var.
5	0,03	144	2084	48	1803	-37	1577	-178	1234	-236	1104	-288	996
6	0,04	285	2558	159	2156	51	1841	-124	1385	-194	1220	-256	1084
7	0,05	430	3062	272	2513	138	2096	-72	1518	-155	1317	-226	1156
8	0,09	580	3499	387	2874	226	2341	-22	1635	-17	1398	-199	1213
9	0,13	736	4169	503	3240	312	2578	25	1737	-83	1467	-173	1259
10	0,18	898	4774	620	3609	398	2806	71	1827	-50	1524	-150	1296
11	0,16	1065	5417	740	3982	483	3026	114	1906	-19	1572	-128	1326
12	0,14	1239	6099	860	4359	567	3238	155	1976	10	1613	-108	1350
13	0,10	1418	6828	982	4742	650	3442	195	2037	37	1647	-89	1369
14	0,05	1604	7602	1106	5130	733	3640	232	2090	62	1676	-72	1384
15	0,03	1796	8428	1231	5524	815	3831	267	2138	85	1700	57	1397

Mean in units of 10^4; variance in units of 10^{10}. Source: Wagle (1967:28)

of results illustrating the outcome of this calculation, given different discount rates and different life times of an investment, appears in Table 5, which also gives the probability distribution of the life of the investment.

Indeed, from Table 5 and by using equations (5) and (6) it is possible to obtain the mean and variance of the conditional distribution of the net present value for different discount rates and the particular life of the investment.

Calculation of the mean and standard deviation of the net present value for different discount rates and the cumulative distribution function of the internal rate of return, for various life times of an investment

With the aid of equations (5) and (6) it is possible to calculate the probability distribution — in terms of means and variances — of NPV_n, given different discount rates and for different life times of the investment. To this end the calculations yield the parameters of the distribution of NPV_n. Likewise equation (9) can be used to obtain the cumulative distribution function of IRR

Table 6 represents a set of hypothetical results portraying the outcome of these calculations, thereby demonstrating the format of these illustrative results.

The intention of the foregoing calculi is to provide a series of solutions and their concomitant results in support of the optimal profitability criterion functions germane to this probabilistic approach, namely, net present value or internal rate of return. The purpose of these criterion functions is twofold, namely

- to convert future income into present income,
- to enable management to understand and evaluate uncertainty.

With these two objectives in mind this article supports the contention that in evaluating capital investment proposals, *cash flow* streams are one of the principal determinants of project worth in the analytic process. However, whilst much of contemporary capital budgeting work is based on assumed 'conditions of certainty', this article advocates the view that probabilistic cash flow formulations afford considerably more insight into the problem of project evaluation and *optimal* selection.

Table 6 Final parametric results relating to the principal investment criterion functions, namely, NPV and IRR

Criterion function	Parameter	Units	Discount rate (% p.a.) 0	2,5	5	10	12,5	15
Net present value	Mean: $E[P(i)]$	10^3	9666	6624	4230	792	−453	−1474
	Second moment: $E[P^2(i)]$	10^{10}	15,900	8885	5036	1993	1591	1532
	Variance: var$[P(i)]$	10^{10}	6556	4497	3247	1930	1570	1315
	SD	10^3	8100	6700	5700	4400	3963	3626
	prob$\{P(i) < 0 \mid i\}$	%	12	16	23	43	54	66
Internal rate of return	Cumulative distributive function		0,12	0,16	0,23	0,43	0,54	0,66

Source: Wagle (1967:29)

Probabilistic formulations, however, introduce some additional problems not encountered in the deterministic case (Bussey & Stevens, 1972:1). For instance, where the periodic cash flow increments comprising the project cash flow stream become *random variables*, it is not only possible for some or all of the increments to be correlated with each other, but it is also a common phenomenon occurring in practice. In such a case the cash flow stream is said to be autocorrelated.

Indeed, the recognition of this phenomenon justifies the *application* of a probabilistic approach to capital investment proposals, incorporating specific real life data and exploring the adequacy of estimating techniques to obtain realistic estimates of the resultant correlation parameters.

To this end it can be shown how the exact mean and variance of the probability distribution of NPV_n and IRR associated with a proposed capital investment project — incorporating the formulation of random cash flow streams based on empirically representative data — can be derived. This application will, moreover, take cognizance of the interregional locational disparities peculiar to locational feasibility studies in South Africa as expressed in the form of *regionally differentiated cash-based decentralization incentives*.

These probability distributions can, it is submitted, provide management with valuable information in analysing the consequences (riskiness) of a proposed *regionally decentralized* investment. Management may also use these results (in particular the mean and the variance of the NPV_n) in ranking such investment proposals. This is because the mean and variance of the NPV_n distribution are most important in the evaluation of a series of *cash flows*, which are peculiar to capital investment proposals in the context of regional decentralization.

Interregional investment comparisons

The rationale underlying *interregional comparisons* is grounded in the principles of sensitivity analysis which is an integral part of risk analysis. Consequently, in so far as it applies to the present article, risk analysis consists of estimating the probability distribution of each factor (variable) affecting an investment decision and then simulating the possible combinations of the values for each variable to determine the range of possible outcomes and the probability associated with each possible outcome, as expressed in terms of NPV and IRR. In effect, Appendix A partially reflects the application of this technique, given the locational constraints peculiar to the industrial development points of Isithebe in Region E. The range estimates in Appendix A purport to represent the *standard* against which interregional comparisons can be made.

Consequently, introducing the notion of risk raises the question of how to assess the element of risk in making comparisons. Therefore, such an assessment of risk must, of necessity, reflect the desirable properties of the variance (σ^2) and, more particularly, its square root, the *standard deviation* (σ) as a standard measure of dispersion. This is purported to be readily interpreted by management whilst being consistent mathematically with probability theory. Indeed, it is submitted, with statistics of this sort, management is able to assess the risk-return trade-off of the project and reach a more sound decision.

The reason for this is that the spread or variability of a risk profile, indicative of the distributional form of NPV, can be measured by the size of the standard deviation which represents the spread around the expected value (μ) of this profitability criterion. Moreover, the corresponding z scores relativize the fluctuations in the means (μ) from one period to the next, in terms of the standard deviation (σ).

Allied to this observation is that the expected return on investment of the proposed project — along with the standard deviation of the financial results obtained — in terms of NPV, will indicate the 'efficiency' of the investment project contemplated (Hertz, 1964:103).

Consequently, what is proposed is a method whereby management can *assess* the risk relating to alternative investment proposals, primarily in terms of the mean (μ), standard deviation (σ), and the corresponding z scores of the probability distribution of NPV, for the required period(s), denoted by n, so as to evaluate the consequences of the different possible outcomes.

Therefore, this application purports to facilitate the choice between a range of alternative localities, as to where a proposed *manufacturing plant* — representing a capital expenditure — could be sited, given the interregional dissimilarities recognized and catered for in terms of South Africa's revised regional economic development programme with its concomitant decentralization inducements.

The corollary to risk analysis is sensitivity analysis. To this end, the purpose of sensitivity analysis is to gauge the influence of each variable, given the region concerned, on the probable outcome of the investment proposal, thereby focusing on those variables, and their respective range values, which are most critical to the locational decision-making process, on an interregional basis (Mirrilees, 1983:213).

Consequently, sensitivity analysis assigns equally likely variations to the value of each affected variable so as to determine the resultant effect on the outcome of the proposal in terms of NPV and IRR. Indeed, a comparison of the values assigned to variables which are deemed to be sensitive to locational disparities highlights the merits of this technique with its emphasis on *investment decision-making* and its underlying locational connotations and assumptions.

Therefore specifically, sensitivity analysis tests the responsiveness of the realities of a locational decision model to possible variations in parameter values, thereby offering valuable information for appraising the relative risk among alternative courses of action.

In essence the resultant means-variance analysis allows for the presentation of a variety of decision outcomes, predominantly in terms of NPV and IRR decision criteria. The purpose of this is to focus attention on variations, if any, in the probability distributions of NPV and IRR, as dominant profitability criterion functions, in the event of a decision to locate a proposed project at an industrial development/deconcentration point in one of the alternative development regions. Indeed, comparative results of this nature, will, it is submitted, constitute an invaluable precedent for the purpose of locational decision-making exercises in the future.

Therefore on the basis of the range estimates, which appear in Appendix A, it is proposed to demonstrate the application of the means-variance approach for the purpose of ascertaining the viability of capital investment proposals relative to South Africa's revised regional development programme.

The purpose of this application, which can be repeated for all the development regions in South Africa, is to show how the exact mean and variance of the probability distribution of the NPV can be derived. The corollary to this approach is the derivation of the approximate probability distribution

Table 7 NPV – Excluding salvage value treating *k* as a variable — Isithebe

Period	μ	σ	z
0	−0,2386000000D + 07		0,90741D + 02
1	−0,2075572532D + 07	0,9088569652D + 04	0,44156D + 03
2	−0,1746364710D + 07	0,1316207940D + 05	0,30582D + 03
3	−0,1391206487D + 07	0,1623430350D + 05	0,26994D + 03
4	−0,1016140396D + 07	0,2052276236D + 05	0,24333D + 03
5	−0,6211562060D + 06	0,2939150568D + 05	0,21018D + 03
6	−0,2279495344D + 06	0,3055221926D + 05	0,19275D + 03
7	0,1073049315D + 06	0,3157519666D + 05	0,17630D + 03
8	0,4238046800D + 06	0,3363640446D + 05	0,16970D + 03
9	0,7558923249D + 06	0,3491056130D + 05	0,16745D + 03
10	0,1089420288D + 07	0,4354997765D + 05	0,15976D + 03
11	0,1430031148D + 07	0,4467940919D + 05	0,15459D + 03
12	0,1776858945D + 07	0,4558253469D + 05	0,15434D + 03
13	0,2112831916D + 07	0,4751149138D + 05	0,15103D + 03
14	0,2437742063D + 07	0,4811745701D + 05	0,14805D + 03
15	0,2748261460D + 07	0,5510959321D + 05	0,14244D + 03
16	0,3048883609D + 07	0,5613821831D + 05	0,13821D + 03
17	0,3339319149D + 07	0,5811179539D + 05	0,13595D + 03
18	0,3616934994D + 07	0,6002226287D + 05	0,13323D + 03
19	0,3883550761D + 07	0,6034917382D + 05	0,13132D + 03
20	0,4134441286D + 07	0,6557794299D + 05	0,12815D + 03
21	0,4376261657D + 07	0,6623324812D + 05	0,12594D + 03
22	0,4607645228D + 07	0,6883925686D + 05	0,12422D + 03
23	0,4827735697D + 07	0,7093752810D + 05	0,12227D + 03
24	0,5035610052D + 07	0,7127659169D + 05	0,12067D + 03
25	0,5229240781D + 07	0,7509243293D + 05	0,11870D + 03
26	0,5413357715D + 07	0,7551187313D + 05	0,11729D + 03
27	0,5592050606D + 07	0,7781100217D + 05	0,11648D + 03
28	0,5755091946D + 07	0,8012245850D + 05	0,11460D + 03
29	0,5899995093D + 07	0,8054882089D + 05	0,11275D + 03
30	0,6033278603D + 07	0,8325345669D + 05	0,11151D + 03
31	0,6154661213D + 07	0,8358278055D + 05	0,11029D + 03
32	0,6262070135D + 07	0,8524010522D + 05	0,10900D + 03
33	0,6361026029D + 07	0,8729174642D + 05	0,10811D + 03
34	0,6441720897D + 07	0,8776962640D + 05	0,10652D + 03
35	0,6512670258D + 07	0,8961189378D + 05	0,10566D + 03
36	0,6588430027D + 07	0,8993750576D + 05	0,10439D + 03
37	0,6611324783D + 07	0,9105751425D + 05	0,10335D + 03
38	0,6636605893D + 07	0,9258067379D + 05	0,10195D + 03
39	0,6645717378D + 07	0,9306528185D + 05	0,10069D + 03
40	0,6646354345D + 07	0,9432215399D + 05	0,10005D + 03
41	0,6624142770D + 07	0,9439185403D + 05	0,98335D + 02
42	0,6565432097D + 07	0,9467548228D + 05	0,97104D + 02
43	0,6529253891D + 07	0,9533233445D + 05	0,95819D + 02
44	0,6452482794D + 07	0,9564013466D + 05	0,94315D + 02
45	0,6356934358D + 07	0,9662599895D + 05	0,92972D + 02
46	0,6243025637D + 07	0,9717084843D + 05	0,91688D + 02
47	0,6111222890D + 07	0,9809531160D + 05	0,90454D + 02
48	0,5964603094D + 07	0,9934598036D + 05	0,89525D + 02

Table 8 NPV, Including salvage value, treating *k* as a variable — Isithebe

Period	μ	σ	z
0			0,90119D + 02
1	0,1740406688D + 06	0,1102021255D + 05	0,25793D + 03
2	0,3746579221D + 06	0,1556596197D + 05	0,20519D + 03
3	0,6005758842D + 06	0,1885834692D + 05	0,19566D + 03
4	0,6693319445D + 06	0,2302886139D + 05	0,18760D + 03
5	0,1156540216D + 07	0,3136694990D + 05	0,17378D + 03
6	0,1448131572D + 07	0,3260079979D + 05	0,16443D + 03
7	0,1667579176D + 07	0,3363283716D + 05	0,15112D + 03
8	0,1918828505D + 07	0,3560507426D + 05	0,14721D + 03
9	0,2160669123D + 07	0,3681127622D + 05	0,14722D + 03
10	0,2413698315D + 07	0,4506938396D + 05	0,14352D + 03
11	0,2678800377D + 07	0,4612932985D + 05	0,14108D + 03
12	0,2954246988D + 07	0,4696223646D + 05	0,14184D + 03
13	0,3222919007D + 07	0,4878837172D + 05	0,13967D + 03
14	0,3484375210D + 07	0,4932551200D + 05	0,13769D + 03
15	0,3735067768D + 07	0,5611760612D + 05	0,13355D + 03
16	0,3979282850D + 07	0,5707724240D + 05	0,13051D + 03
17	0,4216535623D + 07	0,5896942708D + 05	0,12691D + 03
18	0,4444008696D + 07	0,6080447629D + 05	0,12686D + 03
19	0,4663347912D + 07	0,6107951329D + 05	0,12545D + 03
20	0,4869664268D + 07	0,6620763941D + 05	0,12290D + 03
21	0,5069458387D + 07	0,6681542444D + 05	0,12124D + 03
22	0,5261217982D + 07	0,6936121491D + 05	0,11991D + 03
23	0,5443949427D + 07	0,7140848550D + 05	0,11836D + 03
24	0,5616600243D + 07	0,7171148944D + 05	0,11706D + 03
25	0,5777020859D + 07	0,7547487155D + 05	0,11541D + 03
26	0,5929826007D + 07	0,7586355263D + 05	0,11428D + 03
27	0,6078996935D + 07	0,7812615404D + 05	0,11370D + 03
28	0,6214203826D + 07	0,6040468121D + 05	0,11206D + 03
29	0,6332863569D + 07	0,8080732743D + 05	0,11041D + 03
30	0,6441403784D + 07	0,8348350819D + 05	0,10934D + 03
31	0,6539457458D + 07	0,8379329441D + 05	0,10829D + 03
32	0,6624970952D + 07	0,8542955002D + 05	0,10714D + 03
33	0,6703088705D + 07	0,6746139770D + 05	0,10640D + 03
34	0,6764230847D + 07	0,8792413936D + 05	0,10493D + 03
35	0,6816745144D + 07	0,8975039574D + 05	0,10418D + 03
36	0,6855123618D + 07	0,9006369513D + 05	0,10302D + 03
37	0,6881630616D + 07	0,9117139669D + 05	0,10207D + 03
38	0,6891460711D + 07	0,9268294242D + 05	0,10076D + 03
39	0,6886004381D + 07	0,9315810494D + 05	0,99585D + 02
40	0,6872906246D + 07	0,9440566166D + 05	0,99012D + 02
41	0,6837744684D + 07	0,9448789076D + 05	0,97367D + 02
42	0,6786324263D + 07	0,9474451883D + 05	0,96194D + 02
43	0,6719134233D + 07	0,9539473499D + 05	0,94965D + 02
44	0,6631509342D + 07	0,9569671486D + 05	0,93515D + 02
45	0,6525727527D + 07	0,9667691558D + 05	0,92224D + 02
46	0,6402170382D + 07	0,9721685833D + 05	0,90988D + 02
47	0,6261270724D + 07	0,9813670841D + 05	0,89800D + 02
48	0,6106074000D + 07	0,9988290497D + 05	0,88917D + 02

of NPV and IRR. In applying this approach to empirical data *vis-à-vis* Isithebe in Region E, it is the author's intention to generate a *normative* set of decision results against which comparative results, flowing from the application of this technique to the other regions, can be evaluated. These normative results are illustrated in Tables 7 – 10 respectively. In each instance they reflect the results peculiar to 48 financial periods (quarters), with the notable focus being on the 29th and 41st periods, inasmuch as the cash-based labour incentive and the interest and rental subsidies expire at the end of the 28th and 40th periods respectively.

The *raison d'être* of this methodology is to demonstrate how a more accurate appraisal of a potentially risky investment can be effected, given the *disparate locational* constraints peculiar to each of the regions under consideration. Indeed, this is particularly applicable when the risks associated with a proposed investment are perceived to be so formidable that failure to achieve expectations could significantly affect the financial position of the individual or firm.

Such an approach, therefore, provides the locational decision-maker with more comprehensive and objective information indicative of the measure of risk involved in the proposed investment. Moreover, according to the tenor of this article, the nature of the risk involved is a reflection of South Africa's industrial decentralization and regional economic development programme, as partially typified by the third schedule of concessions.

Therefore, on this basis the means-variance approach is able to generate an explicit and comprehensive description of the risk involved in terms of the probability distributions of NPV and IRR. This information, it is contended, permits management (locational decision-makers) to weigh precisely the possible consequences of the proposed investment, thereby making a more accurate decision regarding the proposal.

Conclusions

Noteworthy evidence relating to the application of the proposed, means-variance approach, in the form of a locational decision model, has been elicited by the author. This is the result of the model having been applied to eight development regions in South Africa, with the aid of empirical data peculiar to a locational feasibility study.

To this purpose the application provides a norm against which interregional comparisons can be made.

The results of the application are as follows: The interpretation thereof focuses on the critical periods following the use of the tax allowances, the expiration of the non-taxable cash-based labour incentive at the end of the 28th period and the taxable cash-based interest and rental concessions at the end of the 40th period.

- The cumulative NPV — excluding salvage value — reflects perceptible interregional disparities, which are manifested in the use of the tax allowances and the expiration of the labour incentive. Likewise, similar disparities are evident upon the expiration of the interest and rental concessions.
- The cumulative NPV — including salvage value — proffers further evidence of the impact of the tax allowances, and the corresponding concessions. The means and variances of this NPV, as typified by the corresponding z scores, increase at an exponential rate. This exponential phenomenon attenuates after both the 28th and 40th periods. The magnitude of this exponential variation differs interregionally, albeit marginally.
- The cumulative distribution of IRR, which purports to measure the probability of NPV $\leqslant 0$, in a given period is construed as a further merit of investment worth, relative to the appropriate tax shields and decentralization concessions. However, it is noteworthy that the results thereof are similar for all the regions.

Table 9 Cumulative distribution function of IRR — derived from NPV (excluding salvage value) — where k is treated as a constant — Isithebe

Criterion function	Parameter	Units	Discount rate (k) — percentage per annum: 10, $i=29$	10, $i=41$	12, $i=29$	12, $i=41$	14, $i=29$	14, $i=41$	16, $i=29$	16, $i=41$	18, $i=29$	18, $i=41$
Net present value	Mean $E[NPV_n]$	10^3	7534,56	8667,36	6935,71	7912,74	6393,54	8667,36	5901,38	6634,03	5044,73	5599,68
	Standard deviation	10^2	596,08	708,03	564,66	659,09	536,39	616,30	510,88	578,72	466,82	516,14
	Prob$\{NPV_n < 0/k\}$	%	0	0	0	0	0	0	0	0	0	0
Internal rate of return	Cumulative distribution function		0	0	0	0	0	0	0	0	0	0

Table 10 Cumulative distribution function of IRR — derived from NPV (including salvage value) — where k is treated as a constant — Isithebe

Criterion function	Parameter	Units	Discount rate (k) — percentage per annum: 10, $i=29$	10, $i=41$	12, $i=29$	12, $i=41$	14, $i=29$	14, $i=41$	16, $i=29$	16, $i=41$	18, $i=29$	18, $i=41$
Net present value	Mean $E[NPV_n]$	10^3	8170,75	9035,51	7493,98	8218,8	6884,59	7533,8	6334,25	5787,19	6383,27	5750,59
	Standard deviation	10^2	596,08	708,03	564,66	659,09	536,39	616,28	510,88	578,72	466,82	516,14
	Prob$\{NPV_n < 0/k\}$	%	0	0	0	0	0	0	0	0	0	0
Internal rate of return	Cumulative distribution function		0	0	0	0	0	0	0	0	0	0

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Appendix A Range estimates on basic input variables — Isithebe

				Uniform rate of increase (volume-related)			Number of time periods	Exponential rate of increase (inflation-related)		
Variable	L	Ø	P	L	Ø	P	N	L	Ø	P
S1.S2	30543,5	31154,4	29932,6	1,053	1,067	1,039	12	1,00	1,00	1,00
S3	1,0	1,015	1,005	1,01	1,015	1,005	48	1,0	1,0	1,0
S4	0,042	0,044	0,041	1,0	1,0	1,0	48	1,032	1,044	1,025
V1	– 626000,0	– 679210,0	– 576960,0	1,0298	1,0381	1,0366	48	1,043	1,05	1,038
V2	– 56498,0	– 57910,0	– 55120,0	1,0466	1,0539	1,0310	48	1,05	1,056	1,044
V3										
V4										
V5	– 7500,0	– 8063,0	– 6977,0	1,0444	1,0460	1,00149	48	1,038	1,044	1,032
V6	0,99	0,99	0,99	1,0	1,0	1,0	48	1,0	1,0	1,0
V7	– 50570,0	– 53098,0	– 48162,0	1,0257	1,0398	1,0215	48	1,04	1,048	1,033
V8	– 5460,0	– 5733,0	– 5200,0	1,0419	1,0577	1,0356	48	1,038	1,044	1,032
V9	– 12500,0	– 13125,0	– 11905,0	1,0333	1,0302	1,0348	48	1,05	1,056	1,038
V10	– 34500,0	– 36225,0	– 32775,0	1,0374	1,0432	1,0312	48	1,05	1,056	1,043
V11										
V12										
V13	0,5	0,5	0,5	1,0	1,0	1,0	48	1,0	1,0	1,0
V14										
V15	– 183500,0	– 225000,0	– 175000,0	1,0291	1,0389	1,0365	48	1,043	1,048	1,038
F1	– 6167,0	– 6783,0	– 5550,0	1,0	1,0	1,0	48	1,046	1,05	1,038
F2	– 7500,0	– 8063,0	– 7000,0	1,0	1,0	1,0	48	1,033	1,038	1,025
F3	– 1250,0	– 1344,0	– 1163,0	1,0	1,0	1,0	48	1,038	1,044	1,025
F4	– 1250,0	– 1344,0	– 1163,0	1,0	1,0	1,0	48	1,038	1,044	1,025
F5	– 6000,0	– 6450,0	– 5581,0	1,0	1,0	1,0	48	1,038	1,044	1,025
F6	– 4317,0	– 4500,0	– 4500,0	1,0	1,0	1,0	48	1,05	1,056	1,038
F8	– 7636,0	– 7636,0	– 7636,0	1,0	1,0	1,0	40	1,0	1,0	1,0
F9	–	–	–	–	–	–	–	–	–	–
F10	– 21250,0	– 22500,0	– 20000,0	1,0	1,0	1,0	48	1,0	1,0	1,0
F11	–	–	–	–	–	–	–	–	–	–
L										
L1										
B										
Ø1	– 2336000,0	– 2336000,0	– 2336000,0	1,0	1,0	1,0	01	0,97848	0,97848	0,97848
Ø2	50000,0	50000,0	50000,0	1,0	1,0	1,0	01	0,97848	0,97848	0,97848
H1	– 175000,0	– 255000,0	– 170000,0	0,0	0,0	0,0	01	1,00985	1,00985	1,00985
H2										
Lb										
Bb										
M	14,8	14,8	14,8	1,0	1,0	1,0	40	1,0	1,0	1,0
Ls										
Bs										
Øs	0,7	0,7	0,7	1,0	1,0	1,0	40	1,0	1,0	1,0
WS	0,7	0,7	0,7	1,0	1,0	1,0	40	1,0	1,0	1,0
CL	105,0	105,0	105,0	1,0	1,0	1,0	28	1,0	1,0	1,0
CF	0,95	0,95	0,95	1,0	1,0	1,0	28	1,0	1,0	1,0
N	49,0	50,0	48,0	1,02	1,02	1,02	28	1,0	1,0	1,0
I_3	30898,7	30898,7	30898,7	1,0	1,0	1,0	40	1,0	1,0	1,0
I_4	10683,8	10683,8	10683,8	1,0	1,0	1,0	40	1,0	1,0	1,0
I_7	15435,0	15435,0	15435,0	1,0	1,0	1,0	28	1,0	1,0	1,0
Rc	30000,0	50000,0	25000,0	0,0	0,0	0,0	01	0,0	1,02	1,02
TX_1	0,55	0,55	0,55	1,0	1,0	1,0	48	1,0	1,0	1,0

Appendix A (Continued)

Variable	L	Ø	P	Uniform rate of increase (volume-related) L	Ø	P	Number of time periods N	Exponental rate of increase (inflation-related) L	Ø	P
TX_2	0,25	0,25	0,25	1,0	1,0	1,0	48	1,0	1,0	1,0
TX_3	0,02	0,02	0,02	1,0	1,0	1,0	48	1,0	1,0	1,0
TX_4	1,075	1,075	1,075	1,0	1,0	1,0	48	1,0	1,0	1,0
T	0,462	0,462	0,462	1,0	1,0	1,0	48	1,0	1,0	1,0
D1										
D2										
D3	0,08333	0,08333	0,08333	1,0	1,0	1,0	48	1,0	1,0	1,0

L = Most likely estimate; Ø = Optimistic estimate; P = Pessimistic estimate; N = Number of financial quarters
Note: Variable designation coincides with Appendix B

Appendix B Key to input variable designations

No.	Variable	Description
1	S1	Market demand } sales volume
2	S2	Market share } sales volume
3	S3	Market growth rate
4	S4	Selling price
5	S5	Price change rate
6	SS	Composite sales cash flow
7	V1	Direct materials
8	V2	Direct labour
9	V3	Training costs — non-rebatable
10	V4	Training costs — rebatable
11	V5	Light power and water
12	V6	Electricity subsidy
13	V7	Printing inks and plates
14	V8	Glue
15	V9	Repairs and maintenance
16	V10	Distribution expenses — rail and road
17	V11	Distribution expenses — harbour
18	V12	Distribution expenses — miscellaneous
19	V13	Rebate on railage and road transportation
20	V14	Harbour rebate
21	V15	Net working capital
22	V16	Average incremental cost change
23	VV	Variable cost cash flow
24	F1	Salaries payable to production management
25	F2	Insurance
26	F3	Telex and telephone
27	F4	Stationery
28	F5	Sundries
29	F6	Wages payable to clerical staff
30	F7	Rates
31	F8	Subsidized rental
32	F9	Unsubsidized rental
33	F10	Administration overheads
34	F11	Interest on debt
35	F12	Average incremental cost change
36	FF	Fixed cost cash flow
37	L	Investment in subsidizable land
38	LL	Investment in non-subsidizable land
39	B	Investment in buildings
40	$Ø_1$	Investment in plant
41	$Ø_2$	Investment in other assets (excluding working capital)

Appendix B (continued)

No.	Variable	Description
42	H1	Investment in housing, financed by the industrialist in year '*i*'
43	H2	Recoupment of housing investment via lease or redemption repayments
44	Lb	Book value of land
45	Bb	Book value of buildings
46	M	Market-orientated rate
47	LS	Subsidy factor on land investment
48	BS	Subsidy factor on buildings
49	ØS	Subsidy factor — other assets
50	W	Wage incentive factor
51	WS	Subsidy factor — net working capital
52	cl	Monetary ceiling on wage subsidy
53	CF	Percentage ceiling factor
54	N	Average number of workers per month
55	I_1	Monetary subsidy on land
56	I_2	Monetary subsidy on buildings
57	I_3	Monetary subsidy on other assets
58	I_4	Monetary subsidy on net working capital
59	I_5	Monetary rental subsidy on land
60	I_6	Monetary rental subsidy on buildings
61	I_7	Monetary wage subsidy
62	I_8	Non-taxable training rebate in monetary terms
63	T	Training rebate factor
64	Rc	Relocation allowance
65	D_1	Depreciation/appreciation on land
66	D_2	Depreciation/appreciation on buildings
67	D_3	Depreciation on other assets
68	D_4	Adjusting factor — housing
69	TX_1	Tax shield — initial allowance on plant
70	TX_2	Tax shield — investment allowance on plant and buildings
71	TX_3	Depreciation allowance on buildings
72	TX_4	Wear and tare allowance on plant
73	T	Corporate tax rate
74	R	Residual value
75	R1	Adjusting factor — residual land valuation
76	Rb	Adjusting factor — residual building valuation
77	RØ	Adjusting factor — residual valuation of other assets
78	Rh	Adjusting factor — residual housing valuation
79	S	Number of shifts (to determine TX_4)
80	UR	Unsubsidized rental in period 41
81	SV	Sales volume per shift, per quarter (to determine TX_4)
82	Dr	Discount rate (*k*)