Empirical testing of the Arbitrage Pricing Theory using data from the Johannesburg Stock Exchange

M.J. Page
Graduate School of Business, University of Cape Town, Rondebosch

In 1976 Stephen A. Ross developed a new theory of securities pricing called the Arbitrage Pricing Theory (APT). According to the APT the return an investor can expect from a share is related to the risk-free rate and numerous other factors rather than just the return on the market as predicted by the Capital Asset Pricing Model (CAPM). Although a considerable amount of empirical research has been carried out into the APT in the United States of America, little appears to have been done in South Africa. In this article empirical research is carried out into the APT using data from the JSE. The research involves both attempting to establish the number of 'priced' factors influencing risky security returns on the JSE and comparing the explanatory ability of the APT and CAPM. Factor analysis is used to establish the number of 'priced' APT factors and regression analysis is used to assess the explanatory ability of the models. The findings suggest that at least two factors determine security returns, rather than just the return on the market as predicted by the CAPM, and that a two-factor APT model has significantly better explanatory powers than the CAPM in an *ex-post* sense. Finally, it is apparent that considerably more empirical research needs to be done if the factors are to be conclusively identified and checked for stability through time.
S. Afr. J. Bus. Mgmt. 1986, 17: 38 – 42

Die Arbitrage-prysbepalingteorie (APT) vir die bepaling van die pryse van sekuriteite is gedurende 1976 deur Stephan A. Ross ontwikkel. Hiervolgens is die opbrengs wat 'n belegger van 'n aandeel kan verwag, verwant aan die risikovrye opbrengs en talle ander faktore, eerder as net die opbrengs op die mark soos voorspel deur die kapitaalbate-prysmodel (KP). Alhoewel heelwat empiriese navorsing oor APT reeds in die Verenigde State van Amerika gedoen is, is daar skynbaar nog weinig op dié gebied in Suid-Afrika gedoen. Hierdie studiestuk doen verslag oor die empiriese navorsing oor APT, gebaseer op data van die JE. Die navorsing behels 'n poging om die aantal 'priced' faktore wat riskante sekuriteitopbrengste op die JE beïnvloed, te bepaal en tref ook 'n vergelyking tussen die verklarende vermoëns van APT en KP. Faktoranalise word gebruik om die aantal 'priced' APT-faktore te bepaal en regressie-analise om die verklarende vermoëns van die modelle te bepaal. Die resultate dui daarop dat ten minste twee faktore die opbrengste van sekuriteite bepaal, eerder as net die opbrengs op die mark soos deur die KP voorspel, en dat die verklarende vermoë van 'n twee-faktor APT-model aansienlik beter is as dié van die KP in 'n *ex-post* benadering. Ten slotte is dit duidelik dat heelwat meer empiriese navorsing gedoen sal moet word ten einde die faktore beslissend te identifiseer en na te gaan vir stabiliteit oor 'n lang periode.
S.-Afr. Tydskr. Bedryfsl. 1986, 17: 38 – 42

Presented at the Conference in Management and Economic Sciences, Cape Town, 29 – 31 October 1985

M.J. Page
Graduate School of Business, University of Cape Town, Rondebosch, 7700 Republic of South Africa

Accepted October 1985

Introduction

Formulated by Stephen Ross in 1976, the Arbitrage Pricing Theory (APT) is based on the same intuition as the Capital Asset Pricing Model (CAPM), namely the common variability of asset returns. The theory is, however, far more general than the CAPM and is seen by some researchers to offer a testable alternative to the CAPM (Copeland & Weston, 1983: 211).

According to the Arbitrage Pricing Theory (APT) the return an investor can expect from a share is related to the risk-free rate (time value of money) and numerous other factors rather than just the return on the market as postulated by the CAPM. A further theoretical difference between the APT and the CAPM is that the formulation of the APT does not rely on the existence of market equilibrium but on the absence of arbitrage opportunities. The absence of arbitrage opportunities being a necessary but insufficient condition for market equilibrium (Cho, 1984: 1485).

One of the major advantages of the APT from an empirical research point of view is that the market portfolio does not need to be identified or established as mean-variance efficient. This implies that one need not measure the entire universe of assets but can undertake empirical research using a subset of risky assets. The market portfolio therefore does not play the central and pivotal role in the APT that it does in the CAPM.

The aim of this article is to outline the results of empirical research carried out into the Arbitrage Pricing Theory using data from the Johannesburg Stock Exchange. The research is divided into two sections. In the first section the number of 'priced' factors affecting security returns, in terms of the model, is established. In the second section the Capital Asset Pricing Model and Arbitrage Pricing Theory are compared empirically in order to try and establish conclusive evidence that the APT is a better model for explaining the variability in share returns.

Development of the Arbitrage Pricing Theory

The APT assumes that the rate of return on any asset is a linear function of k factors as shown below:

$$R_i = E(R_i) + b_{i1}F_1 + b_{i2}F_2 + \ldots + b_{ik}F_k + e_i$$

where $R_i =$ the random rate of return on the ith asset; $E(R_i) =$ expected rate of return on the ith asset; $b_{ik} =$ the sensitivity of the ith asset's returns to the kth factor; $F_k =$ the mean zero kth factor common to the returns of all assets under consideration; and $e_i =$ a random zero mean noise term for the ith asset (the unsystematic risk component).

The derivation of the APT is based on choosing a welldiversified arbitrage portfolio. The arbitrage portfolio must be constructed in such a way as to contain no systematic or unsystematic risk, under the following assumptions (Copeland & Weston, 1983: 211 – 218):

(i) Markets are perfectly competitive and frictionless.
(ii) Individuals are assumed to have homogeneous beliefs that the random returns for the set of assets being considered are generated by the linear k-factor model.
(iii) Investors are risk averse.

Based on the above assumptions it can be shown mathematically that, if no arbitrage opportunities exist, any constructed 'arbitrage' portfolio must give zero return and as a consequence the expected return of the ith asset must be a linear function of some constant vector and the sensitivity of that asset's returns to the k factors. Written mathematically:

$$E(R_i) = L_0 + L_1 b_{i1} + L_2 b_{i2} + \ldots + L_k b_{ik}$$

If a risk-free asset exists with return R_f then $L_0 = R_f$. Even if no risk-free asset exists however, L_0 is the common return on all 'zero beta' assets (assets having $b_{ij} = 0$ for all j).

A natural interpretation of L_k is that it represents the risk premium in equilibrium for the kth *factor ($L_k = R_k - R_f$).* In the light of the above the expected return equation can be rewritten in excess returns form as:

$$E(R_i) - R_f = E(R_1 - R_f)b_{i1} + E(R_2 - R_f)b_{i2} + \ldots + E(R_k - R_f)b_{ik}$$

where R_j = return on factor j ($j = 1,2,3. . .,k$); and R_f = return on the risk-free asset.

Review of previous studies

In recent years, considerable research has been carried out in the United States of America into the Arbitrage Pricing Theory but, to date, little research appears to have been done in South Africa.

The empirical research carried out by Roll and Ross is probably one of the major works examining the APT. It was their opinion that the APT offered a testable alternative to the CAPM and, using data from the New York Stock Exchange and the American Stock Exchange, they found that between 1962 and 1972 at least three and probably four 'priced' factors were evident in the return generation process (Roll & Ross, 1980: 1073 – 1103).

Reinganum, in his empirical research, came to the conclusion that a parsimonious APT model could not account for certain of the empirical anomalies that arise within the CAPM. His work involved the examination of the small firm effect on the New York and American Stock Exchanges. He concluded that there was no reason to use the more complicated APT as an empirical replacement of the CAPM, because it failed to account for differences in average returns. However, he did add the rider that in his research several component hypotheses were being jointly tested (Reinganum, 1981: 313 – 320).

The basic issue of the testability of the APT has been debated by several researchers. Shanken expressed the opinion that the usual empirical formulation rules out the very expected return differentials that the theory attempts to explain. However, he felt that an extention of the APT might be testable if observations on the true market portfolio could be found (Shanken, 1982: 1129 – 1140).

In order to check the consistency and stability of the factors that generate daily returns, in terms of the APT, Cho attempted to establish the number of factors in two groups of shares by using interbattery factor analysis, a technique that constrains the factors to be the same across the two groups. He found that there seem to be five or six factors common to both groups (Cho, 1984: 1485 – 1501).

Although, as stated above, little work appears to have been done in South Africa on the APT, Gilbertson and Goldberg made a significant step in the direction of the APT with their empirical research into the JSE using a two-factor model. They proposed that, because of the international impact on the mining sector, the returns of the mining shares will at times be influenced by different 'underlying factors' to the industrial shares. As such they felt that the Market Model should be reformulated to incorporate at least two factors, a 'mining factor' and an 'industrial factor'. Their findings showed that in terms of the reformulated Market Model the returns on shares are a function of their degree of co-movement with the mining and industrial sectors (Gilbertson & Goldberg, 1981:40 – 42).

Data selection

The data base used for the empirical research consisted of weekly and monthly share prices for 200 companies quoted on the Johannesburg Stock Exchange between February 1973 and January 1982. Of these companies 47 were mining companies, 22 were mining finance companies, 17 were financial companies and 114 were industrial companies. One hundred and twenty of the companies were then extracted from the data base and randomly placed in one of four groups subject to the proviso that there were no periods longer than two weeks with zero trading. Table 1 shows the distribution of shares in each of the four groups.

Table 1 Sector representation

Sector	Group 1	Group 2	Group 3	Group 4
Mining	6	7	3	12
Mining finance	3	2	4	3
Financial	2	3	2	2
Industrial	19	18	21	13

Research methodology

The weekly and monthly returns were calculated ignoring dividends and by assuming continuous compounding. The annualized form of the return equation used is given by:

$$R_{i,t} = 365 \times (\log_e P_{i,t}/P_{i,t-1})/N$$

where $R_{i,t}$ = return on the ith asset in time period t; and N = number of days from $t-1$ to t.

The first stage of the empirical research involved a stepped approach in order to identify the number of 'priced' factors. The same methodology was employed for each of the groups on both the weekly and monthly data. The steps are listed and discussed below.

(i) Construction of a correlation matrix from the time series of returns. Pearson product-moment correlation was employed because this technique computes the correlation between pairs of variables (shares) with no controls for the influence of other variables (shares) being made (Nie, Hull, Jenkins, Steinbrenner & Bent, 1975: 276 – 286). The formula for the Pearson correlation coefficient, r, is given by:

$$r = \Sigma(X_i - X)(Y_i - Y) \;/\; [\Sigma(X_i - X)^2 \;\Sigma(Y_i - Y)^2]^{0,5}$$

where X_i = ith return of share X; Y_i = ith return of share Y; X = mean return of share X; and Y = mean return of share Y.

(ii) Principal component factor analysis was then carried out on the correlation matrix to establish the orthogonal factors that, in decreasing order of importance, explained most of the variance in the data. Each factor, or principal component, being a linear combination of the variables (shares) (Harman, 1976: 163 – 184).

(iii) The initial method employed to identify the number of significant factors was the 'scree' test. This involved plotting the eigenvalues against factor numbers and establishing at which factor number the slope of the curve changed. This was then taken to be the least significant factor. (A more rigorous method would have been to use Rippe's test but this was felt to be unnecessary in the light of step iv.)

(iv) Confirmation of the number of significant factors and an estimation of the number that were 'priced' was done by using cross-sectional regressions. For each period the share returns were regressed against the b_{ij}'s obtained from the factor analysis in order to estimate the risk premia associated with the factors and to check whether or not they were significantly different from zero at the 95% level of significance. The method of regression used was ordinary least squares regression and the coefficients (the risk premia being the coefficients in this instance) were tested for significance using Student's t test. Once this had been done for all the periods the percentage of risk premia significantly different from zero for each factor was computed and compared against the percentage expected on the basis of the significance level chosen, namely 95%.

(v) The number of factors estimated from the scree test was then compared against the number found to be 'priced' by virtue of the cross-sectional regression test.

The second stage of the empirical research involved comparing, on an *ex-post* basis, the Arbitrage Pricing Theory and the Capital Asset Pricing Model. Only the four groups of monthly returns were used for this stage of testing. The CAPM itself was first tested by using the Market Model and regressing the excess return on the shares against the excess return on the market, where the return on the market was calculated using the JSE Actuaries overall index. The form of the equation used is given by:

$$R_i - R_f = a_i + b_i(R_m - R_f) + e_i$$

where R_i = the return on the ith asset; R_f = the risk-free rate of return; and e_i = the random error term.

If the CAPM holds true in an *ex-post* sense one would expect the value of a_i not to be significantly different from zero for any of the shares whilst the value of b_i would be different from zero for most shares and equal to beta as defined by the CAPM. Once the betas for all the shares in a group had been estimated cross-sectional regression was used to re-estimate the excess return on the market. This was felt necessary to enable a comparison between the APT and CAPM to be made without biasing the results in favour of the APT.

A comparison between the CAPM and APT was made using the three 'models' listed below:

$$R_i = R_f + b_i(R_m - R_f) + e_i \qquad \text{Model 1 (CAPM)}$$
$$R_i = R_f + b_i(R_m^e) + e_i \qquad \text{Model 2 (CAPM)}$$
$$R_i = R_f + b_{i1}(R_1^e) + b_{i2}(R_2^e) + \ldots + b_{ik}(R_k^e) + e_i \qquad \text{Model 3 (APT)}$$

where R_m = return on the market as calculated from the JSE overall index; R_m^e = excess return on the market calculated using cross-sectional regressions; R_j^e = excess return on factor j calculated using cross-sectional regressions (j = 1,2,3. . .k where k is the number of factors found to be significant in the first-stage testing); b_i = beta coefficient found using the Market Model; b_{ij} = factor loading coefficient found from the factor analysis; and e_i = particular model error term for asset i.

For each share three time series of error terms, one for each of the above models, were created. Comparison between the models was carried out using two techniques. In the first the percentage explanation, the R^2, was calculated for each of the models using the following formula (Pindyck & Rubinfeld, 1981: 63):

$$R^2 = 1 - \Sigma e_i^2 / \Sigma (R_i - \Sigma R_i/n)^2$$

The second method of comparing the models involved regressing the share error terms from each model against the risk premia of each of the other models in order to establish whether some of the unexplained error in one model could be explained by one, or both, of the other models. As with the previous method of comparison the regression R^2 was used as the measure of percentage explanation.

Results of the empirical research

The results of the factor analyses for the four groups using both weekly and monthly data are shown in Figure 1 which is a plot of eigenvalues against factor numbers. As can be seen from the figure there appears to be a remarkable consistency across the groups and also within each group when one compares the weekly and monthly results. There is a definite change of slope of the eigenvalue curve at factor three seeming to suggest that, on the basis of the 'scree' pattern, three true factors exist as predictors of risky asset returns.

The results of the cross-sectional regressions used to estimate the number of 'priced' risk premia in each group are shown in Tables 2 and 3 for the weekly and monthly data respectively. Although there is not as clear a cut-off point between 'priced' and 'non-priced' risk premia as the 'scree' pattern would seem to suggest, it is apparent, from both Table 2 and Table 3, that at least two of the factors are 'priced'.

On the basis of the above it was decided to use a two-factor model and assume that, in terms of the APT, two factors are needed to explain risky asset returns on the Johannesburg Stock

Table 2 Weekly data — percentage of cross-section premia estimates significantly different from zero at the 95% confidence level

	Factor							
Group	1	2	3	4	5	6	7	8
1	52,2	42,5	11,2	13,8	11,2	10,8	6,5	8,8
2	57,3	36,9	11,4	8,2	8,0	5,6	9,7	8,4
3	50,9	42,0	8,2	15,1	9,3	12,5	13,6	9,5
4	56,3	39,4	15,5	12,3	14,2	10,6	14,9	13,6

Table 3 Monthly data — percentage of cross-section premia estimates significantly different from zero at the 95% confidence level

	Factor							
Group	1	2	3	4	5	6	7	8
1	69,2	46,7	26,2	26,2	16,8	11,2	12,2	14,0
2	69,2	46,7	19,6	20,6	16,8	14,0	23,4	7,5
3	72,9	46,7	17,8	18,7	13,1	19,6	14,0	14,0
4	73,8	48,6	22,4	26,2	20,6	28,0	19,6	15,0

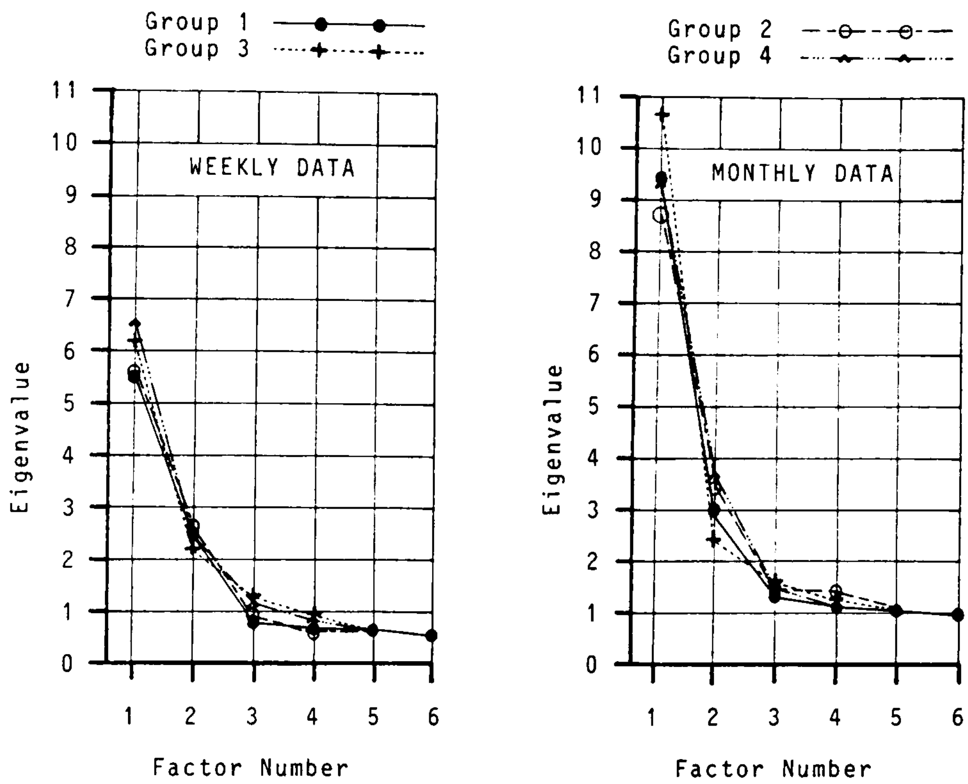

Figure 1 Plots of eigenvalues against factor numbers for all four groups using both weekly and monthly data

Exchange. The APT model selected for comparison with the CAPM is therefore given by:

$$E(R_i) = R_f + b_{i1}E(R_1 - R_f) + b_{i2}E(R_2 - R_f)$$

The *ex-post* testing of the Capital Asset Pricing Model using the Market Model gave the results shown in Table 4. As expected, if the CAPM holds true in an *ex-post* sense, very few of the a_i terms were found to be significantly different from zero (2 out of 120) whilst most of the b_i terms were found to be different from zero (107 out of 120).

The calculation of the R^2 values for the 'three' models, using the time series of error terms for each of the shares, gave the average results shown in Table 5. The two-factor APT model appears to explain a considerably higher percentage of the share return variability than the CAPM, even when the market premium is re-estimated using cross-sectional regressions (107 monthly returns were used for the calculation of each share's R^2).

The results of regressing the two-factor APT model error terms against the market premium of the CAPM showed that practically none of the unsystematic risk, in terms of the APT, could be explained by the CAPM 'factor'. This result applied both when the regression was done against the market premium as originally calculated (Model 1) and when done against the cross-sectional re-estimate of the market premium (Model 2).

When the error terms of the CAPM, as calculated from Model 1, were regressed against the two-factor premia of the APT model a considerable proportion of the error was explained. Regressing the errors against the first-factor premium only showed that even the first factor (the factor explaining most of the variance of the data) explained a significant proportion of the error terms. Regressing the error terms of the CAPM, as calculated from Model 2, against the two-factor premia of the APT model also showed that a significant proportion was explained, although somewhat less than for Model 1. The first factor on its own was, however, unable to explain a significant proportion of the errors.

The percentage explanation of each of the cross model error regressions is shown in summarized form in Table 6 which shows the average R^2 for each of the four groups.

Table 4 Market Model regression results

Percentage different from zero at the 95% level of significance	Group 1	Group 2	Group 3	Group 4
Constant a_i	3	0	0	3
Slope coefficient b_i	90	83	97	87

Table 5 Mean share percentage explanation in each of the groups

Mean R^2	Model 1	Model 2	Model 3
Group 1	0,116	0,268	0,376
Group 2	0,108	0,246	0,379
Group 3	0,124	0,289	0,412
Group 4	0,104	0,262	0,407
Grand mean R^2	0,113	0,266	0,394

Model 1 — standard CAPM estimation
Model 2 — CAPM with the market premium re-estimated
Model 3 — APT two-factor model

Table 6 Cross model error regression results

Error from model	1	1	2	2	3	3
Risk premia from model[a]	3 (1)	3 (1,2)	3 (1)	3 (1,2)	1	2
Group 1	0,144	0,266	0,023	0,191	0,026	0,058
Group 2	0,137	0,280	0,028	0,218	0,026	0,043
Group 3	0,180	0,290	0,039	0,212	0,018	0,043
Group 4	0,113	0,218	0,027	0,170	0,026	0,026
Grand mean R^2	0,143	0,263	0,029	0,198	0,024	0,042

[a]The APT factor premia used in the regressions are shown in brackets where applicable

Conclusions

In the four groups of shares examined two 'priced' factors were consistently evident. This was not immediately apparent from the eigenvalue plots and cross-sectional regressions had to be used. The selection of two factors was ultimately based on the pricing criterion, especially on the weekly results where only two factors were found to be 'priced' to a significant extent. From these results the conclusion is drawn that security returns on the JSE are explained by a two-factor model. The possibility of more 'priced' factors cannot, however, be excluded with certainty.

In comparing the APT and the CAPM, the APT was found to be substantially better with regard to the explanation of variability in share returns. It needs to be re-emphasized though, that these results are *ex-post* findings and not *ex-ante*, and since both models are expectations models the transition from *ex-post* to *ex-ante* deductions has to be made with care. The extent of the difference in explanatory powers by the two models on an *ex-post* basis, however, enables the conclusion to be drawn that the two-factor APT model is the better one.

The question of factor identification was not addressed to any great extent in this research. In the light of the findings of Gilbertson and Goldberg, however, it was felt that it would be instructive to see if there was any similarity between their two-factor Market Model and the factors of the two-factor APT model developed in this article. In addition to using factor analysis to identify the number of 'priced' factors, factor rotation was also employed to get certain of the shares to load heavily on each of the factors. The results of doing this led to one of the 'rotated' factors being composed exclusively of mining related shares whereas the other was a composite of mainly industrial shares for all the groups, on the basis of both the weekly and monthly data.

Although the above would seem to suggest that risky asset returns are a function of how the asset moves relative to the mining and industrial sectors, as suggested by the work of Gilbertson and Goldberg, this is not necessarily the correct deduction to make. What the findings do suggest is that the underlying macro-economic variables determining the return generation process can be divided into those that influence the mining sector to a greater extent and those that effect the industrial sector to a greater extent. Considerably more work needs to done before these variables can be finally identified.

References

Cho, D.C. December 1984. On Testing the Arbitrage Pricing Theory: Inter-Battery Factor Analysis. *J. Fin.*, vol. 39, no. 5.

Copeland, T.E. & Weston, J.F. *Financial Theory and Corporate Policy*. Menlo Park, California: Addison-Wesley Publishing Company, Inc., 1983.

Gilbertson, B. & Goldberg, M. April 1981. The Market Model and The Johannesburg Stock Exchange. *Invest. Anal. J.*, no. 17.

Harman, H.H. *Modern Factor Analysis*. Chicago: University of Chicago Press, 1976.

Nie, N.H., Hull, C.H., Jenkins, J.G., Steinbrenner, Karin & Bent, D.H. *Statistical Package for the Social Sciences*. 2nd ed. New York: McGraw-Hill Book Company, 1975.

Pindyck, R.S. & Rubinfeld, D.L. *Econometric Models and Economic Forecasts*. 2nd ed.: McGraw-Hill International Book Company, 1981.

Reinganum, M.R. May 1981. The Arbitrage Pricing Theory: Some Empirical Results. *J. Fin.*, vol. 36, no. 2.

Roll, R. & Ross, S.A. December 1980. An Empirical Investigation of the Arbitrage Pricing Theory. *J. Fin.*, vol. 35, no. 5.

Shanken, J. December 1982. The Arbitrage Pricing Theory: Is it Testable?. *J. Fin.*, vol. 37, no. 5.